Modeling Regional Interdependencies using a Global Error-Correcting Macroeconometric Model[1]

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Abstract

A financial institution such as a bank is ultimately exposed to macroeconomic fluctuations in the countries to which it has exposure, the most acute example being commercial lending to companies whose fortunes fluctuate with aggregate demand. It was this risk management need for financial institutions which motivated us to build a compact global macroeconometric model capable of generating (point as well as density) forecasts for a core set of macroeconomic factors for a set of regions and countries which explicitly allows for interconnections and interdependencies that exist between national and international factors. This paper provides such a global modeling framework by making use of recent advances in the analysis of cointegrating systems. In an unrestricted VAR(p) model in k endogenous variables covering N countries, the number of unknown parameters will be unfeasibly large, of order $p(kN-1)$, requiring a more parsimonious solution. We first estimate individual country/region specific vector error-correcting models, where the domestic macroeconomic variables are related to corresponding foreign variables constructed exclusively to match the international trade pattern of the country under consideration. The individual country models are then combined in a consistent and cohesive manner to generate forecasts for all the variables in the world economy simultaneously. We estimate the model using quarterly data from 1979Q1 to 1999Q1 and shed light on the degree of regional interdependencies by investigating the time profile of the transmission of shocks of one variable to the rest of the world.

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JEL Classification: C320, E170, G200.

1 Introduction

Over the past decade the world economy has become increasingly globalized with important consequences for the conduct of monetary and financial policies and risk management. In setting interest rates, more than ever before central bankers need to allow for the inter-relationships that exist between their economy and the rest of the world. Moreover, the risk analysis of the financial activities of commercial banks needs to take account of domestic economic conditions as well as the economic conditions of countries that directly or indirectly influence the loss distribution of banks' loan portfolios.

At the heart of any credit risk analysis is a mapping from states of the world economic conditions to the loss (or change in value) distribution of the credit portfolio. Bangia et al. (2000) show that, for the US case, the economic capital required to capitalize a bank during a recession year is about 25-30% higher than during an expansion year. Thus portfolios which are diversified across industries but not across countries remain dangerously tied to that single risk factor called the macroeconomy. International diversification helps. Clark and van Wincoop (2001), comparing the US and 14 EU countries, show that within country correlations of output are far greater than cross-country correlations. Backus and Kehoe (1992) show that output correlations between 10 major industrialized countries have remained very stable over most of the 20th century. The motivation of linking a credit portfolio model to the macroeconomy becomes quite clear: being able to forecast the economy, even with error, will greatly help an institution plan with its risk management strategy. Banks with international portfolios (and this would include most of the top-50 players globally) are therefore highly motivated to have their macroeconomic engine reflect the geographic diversification of their loan portfolio. In short, both commercial and central bankers need to work with a global macroeconometric model which is capable of generating forecasts for a core set of macroeconomic factors for a set of regions and countries to which they have risk exposures and explicitly allow for interconnections and interdependencies that exist between national and international factors in a coherent and consistent manner.

This paper aims to provide such a global modelling framework by making use of recent advances in the analysis of cointegrating systems. So far applications of the cointegrating approach have been confined to a single country covering only some of the key macro-economic variables.[1] While in principle it is possible to extend the approach to model inter-relationships across different economies, in practice due to data limitations such a strategy will not be feasible. In an unrestricted VAR model covering N regions the number of unknown parameters rises with N, and even if we focus on a few key macroeconomic indicators such as output, inflation, interest rate, and exchange rate there will be $p(kN-1)$ unknown parameters (not counting intercepts or other deterministic/exogeneous variables) to be estimated per each equation, where p is the order of the VAR and k is the number of the endogenous variables per region. For example, in the case of a world economy composed of 10 regions with $p=2$, and $k=5$, there will be at least as many as 98 unknown coefficients to be estimated per equation with the available time series being of the same order of magnitude for

[1] See, for example, King, Plosser, Stock, and Watson (1991), Mellander, Vredin, and Warne (1992), Crowder, Hoffman and Rasche (1999), and Garratt, Lee, Pesaran and Shin (2000, 2001).

advanced economies and often much less in the case of other regions.

In view of these difficulties global forecasting models are often formed by linking up of the traditional, often large-scale, macroeconometric models developed originally for the national economies. A prominent example of this approach is Larry Klein's Project Link adopted by United Nations. A similar approach, but on a smaller scale, has been followed by international agencies such as IMF and OECD. The National Institute's Global Econometric Model (NiGEM) estimates/calibrates a common model structure across OECD countries, China and a number of regional blocks. The country/region specific models in NiGEM are still quite large each comprised of 60-90 equations with 30 key behavioural relations. For a recent detailed account see Barrell et al. (2001). Global models with limited geographical coverage have also been developed. For example, Rae and Turner (2001) develop a small forecasting model covering the United States, the Euro area and Japan. These contributions provide significant insights into the important inter-linkages that exist among major world economies and have proved essential in global forecasting. Nevertheless, they are difficult to use for risk management purposes, and do not adequately (for risk analysis) address the important financial inter-linkages that exist amongst the world's major economies.

In this paper we propose a new approach to modelling of the global economy which avoids some of these limitations, while at the same time providing a consistent and flexible framework for use in a variety of applications such as risk management. We first estimate individual country (or region) specific vector error-correcting models (VECM) where the domestic macro-economic variables such as Gross Domestic Product (GDP), the general price level, the level of short-term interest rate, exchange rate, equity prices (when applicable) and money supply, are related to corresponding foreign variables constructed exclusively to match the international trade pattern of the country under consideration. For purposes of estimation and inference these country-specific foreign variables can be treated as exogenous for most economies when N is sufficiently large; a notable exception of course being the US economy. The model for the US can be estimated by treating most of the variables as endogenous. The individual country models are then combined in a consistent and cohesive manner to generate forecasts for *all* the variables in the world economy simultaneously.

The plan of the paper is as follows: Section 2 sets out the country/region specific models and establishes the inter-linkages between each of the economies and the rest of the world through trade-based weighting matrices. The different country-specific VECM models are then combined in Section 3, where a complete solution of the global VAR (GVAR) model is provided. Section 4 examines the error-correcting properties of the global model and shows that the number of long-run relationships in the global model can not exceed the sum of the long-run relations of the region specific models. Dynamic properties of the GVAR model and its stability properties are discussed in Section 5. Section 6 derives impulse response functions for the analysis of shocks in one country on the macro-economic variables in other countries. Section 7 considers the estimation problem of the country-specific models, with the technical details provided in Appendix (A). Section 8 discusses the practical issues surrounding the construction of regional aggregates. To ensure maximum global coverage while keeping the risk analysis manageable it is often necessary to work at regional levels and Section 8 also addresses the aggregation bias that this may entail and ways of minimizing such a basis. An empirical illustration of the approach is set out in Section 9, where a GVAR model in four countries (US, Germany, China and Japan) and five regions (Western Europe, Central Europe, Middle East, South East Asia, and Latin America) is estimated and analyzed. This section also reports a number of impulse response functions demonstrating how the model could be used in the analysis of the transmission of stock market and interest rate shocks from one region to the rest of the world economy. Section 10 offers some concluding remarks. Appendix (B) provides a summary of data sources used, as well as a brief account of the way regional series were constructed.

2 Country Specific Models

We assume there are $N+1$ countries (or regions) in the global economy, indexed by $i = 0, 1, 2, ..., N$. We adopt country 0 as the reference country. (US seems an obvious choice). For each country we assume that the country specific variables are related to the global economy variables measured as country-specific weighted averages of foreign variables, deterministic variables such as time trends and exogenously determined variables such as oil prices or other raw material prices. Focusing first on the domestic and foreign variables only, we model the relationships for individual economies using the following simple log-linear vector autoregressive specification:[2]

$$\begin{aligned} \mathbf{x}_{it} &= \mathbf{a}_{i0} + \mathbf{\Phi}_i \mathbf{x}_{i,t-1} + \mathbf{\Lambda}_{i0} \mathbf{x}_{it}^* + \mathbf{\Lambda}_{i1} \mathbf{x}_{i,t-1}^* + \boldsymbol{\varepsilon}_{it}, \\ t &= 1, 2, ..., T;\ i = 0, 1, 2, ..., N \end{aligned} \tag{2.1}$$

where $\mathbf{x}_{it}$ is the $k_i \times 1$ country-specific factors/variables, $\mathbf{\Phi}_i$ is a $k_i \times k_i$ matrix of associated lagged coefficients, $\mathbf{x}_{it}^*$ is the $k_i^* \times 1$ vector of foreign variables specific to country i (to be defined below) with $\mathbf{\Lambda}_{i0}$ and $\mathbf{\Lambda}_{i1}$ being $k_i \times k_i^*$ matrix of fixed coefficients, and $\boldsymbol{\varepsilon}_{it}$ is a $k_i \times 1$ vector of country-specific shocks assumed to be serially uncorrelated with a zero mean and a non-singular covariance matrix, $\mathbf{\Sigma}_{ii} = (\sigma_{ii,\ell s})$, where $\sigma_{ii,\ell s} = cov(\varepsilon_{i\ell t}, \varepsilon_{ist})$, or written more compactly[3]

$$\boldsymbol{\varepsilon}_{it} \backsim i.i.d.(\mathbf{0}, \mathbf{\Sigma}_{ii}). \tag{2.2}$$

We also allow for the shocks to be correlated across regions. In particular, we assume that

$$\begin{aligned} E\left(\boldsymbol{\varepsilon}_{it}\boldsymbol{\varepsilon}_{jt'}'\right) &= \mathbf{\Sigma}_{ij} \text{ for } t = t', \\ &= \mathbf{0} \text{ for } t \neq t. \end{aligned}$$

The assumption that the variance covariance matrices, $\mathbf{\Sigma}_{ij}$, $i, j = 0, 1, 2, ..., N$, are time invariant can be relaxed, but for the analysis of quaterly observations this time invariant assumption may not be too restrictive. However, when the

[2]For simplicity, the exposition here is confined to first-order VAR models. But the analysis can be easily extended to higher-order processes. The VAR formulation is also quite general and accommodates many open macroeconometric specifications. For details see, for example, Garratt et al. (2000, 2001).

[3]Deterministic trends and other common global variables, such as oil prices, can also be included in the model. These are not central to our exposition here and will be considered in Section 5.

focus of the analysis is on contagion or spillover effects resulting from systemic risk it may be necessary to consider regime switching models where the parameters of the regional models (in particular $\mathbf{\Sigma}_{ij}$) switch between a "normal" and a "crisis" set of values.[4] To accomodate such effects it would be necessary to specifiy and estimate non-linear switching regional models from which a non-linear global model can be derived, and this is beyond the scope of the present paper.

Typically $\mathbf{x}_{it}$ will include real output (y_{it}), a general price index (p_{it}) or its rate of change, a real equity price index (q_{it}), the exchange rate (e_{it}, measured in terms of a reference currency, say US dollar), an interest rate (ρ_{it}), and real money balances (m_{it}). To focus ideas we set $\mathbf{x}_{it} = (y_{it}, p_{it}, q_{it}, e_{it}, \rho_{it}, m_{it})'$, with $k_i = 6$.[5] We assume that these variables are observed at quarterly frequencies; $y_{it}, p_{it}, q_{it}, e_{it}$, and m_{it} are measured in natural logarithms and ρ_{it} is an interest rate variable. Output could be measured by real gross domestic (or national) product (GDP); the general price level by the consumer price index (CPI), the real equity price index (when available) could be measured by broad market indices such as the Standard and Poor 500 index in the US, or the All Times Share index in the UK, deflated by the CPI, the real money supply by M_0 or M_2 measures of money supply deflated by the CPI, and finally the interest rate variable could be either the nominal interest rate on three months Treasury Bill rate (or its equivalent), or the (ex post) real interest rate defined as the nominal rate minus the rate of inflation.[6] For example, a typical set of endogenous variables for country i ($i \neq 0$), could be:

$$\left.\begin{array}{r}
y_{it} = \ln\left(GDP_{it}/CPI_{it}\right), \\
p_{it} = \ln(CPI_{it}), \\
q_{it} = \ln(EQ_{it}/CPI_{it}), \\
m_{it} = \ln\left(M_{it}/CPI_{it}\right), \\
e_{it} = \ln(E_{it}), \\
\rho_{it} = 0.25 * \ln(1 + R_{it}/100),
\end{array}\right\} \tag{2.3}$$

where[7]

$$\begin{array}{rcl}
GDP_{it} & = & \text{Nominal Gross Domestic Product of country } i \\
 & & \text{during period } t, \text{ in domestic currency,} \\
CPI_{it} & = & \text{Consumer Price Index in country } i \text{ at time } t, \\
 & & \text{equal to 1.0 in a base year (say 1996),} \\
M_{it} & = & \text{Nominal Money Supply in domestic currency,} \\
EQ_{it} & = & \text{Nominal Equity Price Index,} \\
E_{it} & = & \text{Exchange rate of country } i \text{ at time } t \text{ in terms of } US \text{ dollars,} \\
R_{it} & = & \text{Nominal rate of interest per annum, in per cent.}
\end{array}$$

[4] A comprehensive review of the literature on systemic risk can be found in De Bandt and Hartmann (2000).

[5] However, in practice it may be necessary to consider other transformations of these underlying variables. For example, as can be see from our empirical analysis in Section 9, we argue in favour of using the rate of inflation ($p_{it} - p_{i,t-1}$) instead of the price level (p_{it}) and the real exchange rate ($e_{it} - p_{it}$) instead of the nominal exchange rate (e_{it}).

[6] For details of the variables used in our empirical application and their sources see Section 9, and the Data Appendix.

[7] Note that the last transformation specified in (2.3) converts the annual rate of interest, R_{it}, to quarterly interest rate, ρ_{it}, using a logarithmic scale.

Notice, that in the case of the base economy $e_{it} = 0$ and $\mathbf{x}_{0t} = (y_{0t}, p_{0t}, q_{0t}, \rho_{0t}, m_{0t})'$, with $k_0 = 5$. Also in the case of some of the emerging market economies and the newly constituted economies of the Eastern Europe and Russia where the interest rate and/or the equity price index may not be available over the whole sample period, $\mathbf{x}_{it}$ may be confined to the $y_{it}, p_{it}, e_{it}, m_{it}$, with $k_i = 4$. The foreign variables (indices), denoted by $\mathbf{x}^*_{it}$, is a $k^*_i \times 1$ vector[8] are constructed as weighted averages, with country/region specific weights:

$$\left.\begin{array}{c} \mathbf{x}^*_{it} = (y^*_{it}, p^*_{it}, q^*_{it}, e^*_{it}, \rho^*_{it}, m^*_{it})', \\ y^*_{it} = \sum_{j=0}^{N} w^y_{ij} y_{jt}, \quad p^*_{it} = \sum_{j=0}^{N} w^p_{ij} p_{jt}, \\ q^*_{it} = \sum_{j=0}^{N} w^q_{ij} q_{jt}, \quad e^*_{it} = \sum_{j=1}^{N} w^e_{ij} e_{jt}, \\ \rho^*_{it} = \sum_{j=0}^{N} w^\rho_{ij} \rho_{jt}, \quad m^*_{it} = \sum_{j=0}^{N} w^m_{ij} m_{jt}. \end{array}\right\} \tag{2.4}$$

The weights w^y_{ij}, $w^p_{ij}, w^q_{ij}, w^e_{ij}, w^\rho_{ij}$, and w^m_{ij} for $i, j = 0, 1, ...N$,[9] could be based on trade shares (namely the share of country j in the total trade of country i measured in US dollars) in the case of y^*_{it}, p^*_{it}, e^*_{it} and m^*_{it} and capital flows in the case of equity price indices and interest rates, q^*_{it} and ρ^*_{it}.[10] Notice that

$$w^y_{ii} = w^p_{ii} = w^q_{ii} = w^\rho_{ii} = w^m_{ii} = w^e_{ii} = 0, \text{ for all } i.$$

It is worth noting that the exchange rate variable, e^*_{it}, defined for country i is not the same as the more familiar concept of the 'effective exchange rate'. To see this denote the exchange rate of country i in terms of the currency of country j by E_{ijt}. Then

$$\ln(E_{ijt}) = \ln(E_{it}/E_{jt}) = e_{it} - e_{jt}. \tag{2.5}$$

Let the trade share of country i with respect to country j be w^T_{ij} and write the (log) effective exchange rate of country i as (recall that $e_{0t} = 0$):[11]

$$\begin{aligned} \tilde{e}_{it} &= \sum_{j=0}^{N} w^T_{ij}(e_{it} - e_{jt}) \\ &= \left(\sum_{j=0}^{N} w^T_{ij}\right) e_{it} - \sum_{j=1}^{N} w^T_{ij} e_{jt}. \end{aligned}$$

But, $\sum_{j=0}^{N} w^T_{ij} = 1$ and

$$\tilde{e}_{it} = e_{it} - \sum_{j=1}^{N} w^T_{ij} e_{jt},$$

[8] In our application, $k^*_i = 5$ or 6. See Section 9.

[9] In practice, it may also be desirable to allow for these weights to vary over time in order to capture secular movements in the geographical patterns of trade and capital flows. However, too frequent changes in the weights could introduce an undesirable degree of randomness into the analysis. This is the classic index number problem to which a totally satisfactory answer does not exist. In our empirical analysis we use fixed trade weights but base their computation on averages of trade flows over a three year period.

[10] See Glick and Rose (1999) who discuss the importance of trade links in the analysis of contagion.

[11] w^T_{ij} can be measured as the total trade between country i and country j divided by the total trade of country i with *all* its trading partners.

or

$$e_{it}^* = e_{it} - \tilde{e}_{it}.$$

Only in the case of the base country the two concepts coincide (apart from a sign convention):

$$e_{0t}^* = -\tilde{e}_{0t}.$$

It is also worth noting that in the case of countries or regions that attempt to maintain (approximately) a fixed effective exchange rate by pegging their currency to a basket of currencies, there will be a close correlation between e_{it} and e_{it}^* and for purposes of econometric analysis it will not be advisable to include e_{it}^* as an exogenous variable in $\mathbf{x}_{it}^*$, considering that e_{it} is already included amongst the endogenous variables. The inclusion of e_{it} in the model ought to be sufficient to accommodate the possible effects of exchange rate variations on the domestic economy. For the base economy, however, under our set up e_{0t}^* will be determined by the models for the rest of the world via equation (2.1), for $i = 1, 2, .., N$. Hence, for internal consistency e_{0t}^* must be treated as an exogenous variable in the model for the base economy. Otherwise, there will be two sets of equations explaining e_{0t}^*; one equation derived by combining the exchange rate equations from the models for the regions $i = 1, 2, .., N$, and a second equation obtained directly from the model of country $i = 0$, if e_{0t}^* is included in that model as endogenous.

The $N + 1$ country-specific models, (2.1), together with the relations linking the exogenous variables of the country-specific models to the variables in the rest of the global model, (2.4), provide a complete system. As emphasized in the introduction, due to data limitations even for moderate values of N, a full system estimation of the global model is not feasible. To avoid this difficulty we propose to estimate the parameters of the country-specific models separately, treating the foreign price variables as exogenously given on the grounds that most economies (possibly with the exception of the US) are small relative to the size of the world economy. In this approach the accuracy of the approximation is likely to increase with the number of countries under consideration.

Although country-specific models are estimated separately, we nevertheless maintain a general specification for the correlation of shocks across the different countries/regions. This flexibility is particularly important for the simulation of loss distributions. In general, the GVAR model allows for interactions amongst the different economies through three separate but inter-related channels:

1. Direct dependence of $\mathbf{x}_{it}$ on $\mathbf{x}_{it}^*$ and its lagged values.

2. Dependence of the country-specific variables on common global exogenous variables such as oil prices. (see Section 5).

3. Non-zero contemporaneous dependence of shocks in country i on the shocks in country j, measured via the cross country covariances, Σ_{ij}, defined by

$$\Sigma_{ij} = Cov(\boldsymbol{\varepsilon}_{it}, \boldsymbol{\varepsilon}_{jt}) = E(\boldsymbol{\varepsilon}_{it}\boldsymbol{\varepsilon}_{jt}'), \tag{2.6}$$

   where $\boldsymbol{\varepsilon}_{it}$ is defined by (2.1). A typical element of Σ_{ij} will be denoted by $\sigma_{ij,\ell s} = cov(\varepsilon_{i\ell t}, \varepsilon_{jst})$ which is the covariance of the ℓth variable in country i with the sth variable in country j.

3 Solution of the GVAR Model

Due to the contemporaneous dependence of the domestic variables, $\mathbf{x}_{it}$, on the foreign variables, $\mathbf{x}_{it}^*$, the country-specific VAR models (2.1) need to be solved simultaneously for all the domestic variables, $\mathbf{x}_{it}$, $i = 0, 1, ..., N$. The solution can then be used in impulse response analysis (also known as the scenario shock analysis) and for *ex ante* forecasting. For this purpose we first define the $(k_i + k_i^*) \times 1$ vector

$$\mathbf{z}_{it} = \begin{pmatrix} \mathbf{x}_{it} \\ \mathbf{x}_{it}^* \end{pmatrix}, \tag{3.1}$$

and rewrite (2.1) as

$$\mathbf{A}_i \mathbf{z}_{it} = \mathbf{a}_{i0} + \mathbf{B}_i \mathbf{z}_{i,t-1} + \varepsilon_{it}, \tag{3.2}$$

where

$$\mathbf{A}_i = (\mathbf{I}_{k_i}, -\mathbf{\Lambda}_{i0}), \quad \mathbf{B}_i = (\mathbf{\Phi}_i, \mathbf{\Lambda}_{i1}). \tag{3.3}$$

The dimensions of $\mathbf{A}_i$ and $\mathbf{B}_i$ are $k_i \times (k_i + k_i^*)$ and $\mathbf{A}_i$ has a full column rank, namely $Rank(\mathbf{A}_i) = k_i$.

Collect all the country-specific variables together in the $k \times 1$ vector $\mathbf{z}_t = (\mathbf{y}_t', \mathbf{p}_t', \mathbf{q}_t', \mathbf{e}_t', \boldsymbol{\rho}_t', \mathbf{m}_t')'$, where $k = \sum_{i=0}^N k_i$ is the total number of the endogenous variables in the global model and

$$\begin{aligned} \mathbf{y}_t &= (y_{0t}, y_{1t}, ..., y_{Nt})', \ \mathbf{p}_t = (p_{0t}, p_{1t}, ..., p_{Nt})', \\ \mathbf{q}_t &= (q_{0t}, q_{1t}, ..., q_{Nt})', \ \mathbf{e}_t = (e_{1t}, ..., e_{Nt})', \\ \boldsymbol{\rho}_t &= (\rho_{0t}, \rho_{1t}, ..., \rho_{Nt})', \text{ and } \mathbf{m}_t = (m_{0t}, m_{1t}, ..., m_{Nt})'. \end{aligned}$$

In the case where all the six main variables are present across all the countries/regions then $k = 6N + 5$. Notice that $e_{0t} = 0$ and $\mathbf{e}_t$ at most will be N dimensional.

It is now easily seen that the country specific variables can all be written in terms of $\mathbf{z}_t$:

$$\mathbf{z}_{it} = \mathbf{W}_i \mathbf{z}_t, \quad i = 0, 1, 2, ..., N, \tag{3.4}$$

where $\mathbf{W}_i$ is a $(k_i + k_i^*) \times k$ matrix of fixed (known) constants defined in terms of the country specific weights w_{ij}^y, $w_{ij}^p, w_{ij}^q, w_{ij}^e, w_{ij}^\rho$, and w_{ij}^m. $\mathbf{W}_i$ can be viewed as 'link' matrix that allows the country-specific models to be written in terms of the global variable vector, $\mathbf{z}_t$.

Using (3.4) in (3.2) we have:

$$\mathbf{A}_i \mathbf{W}_i \mathbf{z}_t = \mathbf{a}_{i0} + \mathbf{B}_i \mathbf{W}_i \mathbf{z}_{t-1} + \varepsilon_{it},$$

where $\mathbf{A}_i \mathbf{W}_i$ and $\mathbf{B}_i \mathbf{W}_i$ are both $k_i \times k$ dimensional matrices. Stacking these equations now yields:

$$\mathbf{G}\mathbf{z}_t = \mathbf{a}_0 + \mathbf{H}\mathbf{z}_{t-1} + \varepsilon_t, \tag{3.5}$$

where

$$\mathbf{a}_0 = \begin{pmatrix} \mathbf{a}_{00} \\ \mathbf{a}_{10} \\ \vdots \\ \mathbf{a}_{N0} \end{pmatrix}, \ \varepsilon_t = \begin{pmatrix} \varepsilon_{0t} \\ \varepsilon_{1t} \\ \vdots \\ \varepsilon_{Nt} \end{pmatrix}, \tag{3.6}$$

$$\mathbf{G}=\begin{pmatrix}\mathbf{A}_0\mathbf{W}_0\\ \mathbf{A}_1\mathbf{W}_1\\ \vdots\\ \mathbf{A}_N\mathbf{W}_N\end{pmatrix},\ \mathbf{H}=\begin{pmatrix}\mathbf{B}_0\mathbf{W}_0\\ \mathbf{B}_1\mathbf{W}_1\\ \vdots\\ \mathbf{B}_N\mathbf{W}_N\end{pmatrix}. \tag{3.7}$$

It is easily seen that $\mathbf{G}$ is a $k\times k$ dimensional matrix and in general will be of full rank, and hence non-singular. Then the GVAR model in all the variables can be written as

$$\mathbf{z}_t=\mathbf{G}^{-1}\mathbf{a}_0+\mathbf{G}^{-1}\mathbf{H}\mathbf{z}_{t-1}+\mathbf{G}^{-1}\varepsilon_t,$$

which may also be solved recursively forward to obtain the future values of $\mathbf{z}_t$. See Section 5 below for further details.

It is worth illustrating the above solution technique by means of a simple example. Consider a global model composed of three regions in three variables, say output, prices, output exchange rates (all in logs). Then

$$\mathbf{z}_t=\begin{pmatrix}y_{0t}\\ y_{1t}\\ y_{2t}\\ p_{0t}\\ p_{1t}\\ p_{2t}\\ e_{1t}\\ e_{2t}\end{pmatrix},\ z_{0t}=\begin{pmatrix}y_{0t}\\ p_{0t}\\ y^*_{0t}\\ p^*_{0t}\\ e^*_{0t}\end{pmatrix},\ z_{it}=\begin{pmatrix}y_{it}\\ p_{it}\\ e_{it}\\ y^*_{it}\\ p^*_{it}\\ e^*_{it}\end{pmatrix},\ i=1,2.$$

Using the trade shares, w^T_{ij}, to construct the foreign variables and recalling that

$$\begin{aligned} e^*_{0t} &= w^T_{01}e_{1t}+w^T_{02}e_{2t},\\ e^*_{1t} &= w^T_{12}e_{2t},\\ e^*_{2t} &= w^T_{21}e_{1t}, \end{aligned}$$

then the link matrices for these three regions are

$$\mathbf{W}_0=\begin{pmatrix}1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&w^T_{01}&w^T_{02}&0&0&0&0&0\\ 0&0&0&0&w^T_{01}&w^T_{02}&0&0\\ 0&0&0&0&0&0&w^T_{01}&w^T_{02}\end{pmatrix},$$

$$\mathbf{W}_1=\begin{pmatrix}0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&1&0\\ w^T_{10}&0&w^T_{12}&0&0&0&0&0\\ 0&0&0&w^T_{10}&0&w^T_{12}&0&0\\ 0&0&0&0&0&0&0&w^T_{12}\end{pmatrix},$$

$$\mathbf{W}_2=\begin{pmatrix}0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&1\\ w^T_{20}&w^T_{21}&0&0&0&0&0&0\\ 0&0&0&w^T_{20}&w^T_{21}&0&0&0\\ 0&0&0&0&0&0&w^T_{21}&0\end{pmatrix}.$$

Notice that the country-specific weights are non-negative and satisfy the adding up restrictions $w_{01}^T + w_{02}^T = 1$, $w_{10}^T + w_{12}^T = 1$, $w_{20}^T + w_{21}^T = 1$. Furthermore, in the case where trade shares are non-zero it is easily seen that the link matrices are of full row ranks, a property that will be of importance when we come to consider the error-correction properties of the global model in the following section. Finally,

$$\mathbf{A}_0 = (\mathbf{I}_2, -\mathbf{\Lambda}_{00}), \ \mathbf{A}_1 = (\mathbf{I}_3, -\mathbf{\Lambda}_{10}), \ \mathbf{A}_2 = (\mathbf{I}_3, -\mathbf{\Lambda}_{20}),$$

where $\mathbf{I}_s$ is an identity matrix of order s. Using the above $\mathbf{W}_i$ and $\mathbf{A}_i$ matrices the $\mathbf{G}$ matrix defined by (3.7) can now be readily constructed. In this example $\mathbf{G}$ is 8×8 and must be non-singular if the global model is to be complete.[12]

4 Error-correcting Properties of the Global Model

It is of interest to see the extent to which error-correcting properties of the country/region specific models are reflected in the global model, (3.5). Rewriting (2.1) in the error-correction form

$$\begin{aligned} \Delta \mathbf{x}_{it} &= \mathbf{a}_{i0} - (\mathbf{I}_{k_i} - \mathbf{\Phi}_i)\mathbf{x}_{i,t-1} + (\mathbf{\Lambda}_{i0} + \mathbf{\Lambda}_{i1})\mathbf{x}_{i,t-1}^* \\ &\quad + \mathbf{\Lambda}_{i0}\Delta\mathbf{x}_{it}^* + \boldsymbol{\varepsilon}_{it}, \end{aligned} \tag{4.1}$$

and using (3.1)

$$\Delta \mathbf{x}_{it} = \mathbf{a}_{i0} - (\mathbf{A}_i - \mathbf{B}_i)\mathbf{z}_{i,t-1} + \mathbf{\Lambda}_{i0}\Delta\mathbf{x}_{it}^* + \boldsymbol{\varepsilon}_{it}, \tag{4.2}$$

where as before $\mathbf{z}_{it} = (\mathbf{x}_{it}', \mathbf{x}_{it}^{*\prime})'$, and $\mathbf{A}_i$ and $\mathbf{B}_i$ are already defined by (3.3). The error-correction properties of the model for country/region i is summarized in the $k_i \times (k_i + k_i^*)$ matrix

$$\mathbf{\Pi}_i = \mathbf{A}_i - \mathbf{B}_i. \tag{4.3}$$

In particular, the rank of $\mathbf{\Pi}_i$, say $r_i \leq k_i$, specifies the number of "long-run" relationships that exists amongst the domestic and the country-specific foreign variables, namely , $\mathbf{x}_{it}$ and $\mathbf{x}_{it}^*$. Therefore, following Johansen (1991) we have

$$\mathbf{A}_i - \mathbf{B}_i = \boldsymbol{\alpha}_i \ \boldsymbol{\beta}_i', \tag{4.4}$$

where $\boldsymbol{\alpha}_i$ is the $k_i \times r_i$ loading matrix of full column rank, and $\boldsymbol{\beta}_i$ is the $(k_i + k_i^*) \times r_i$ matrix of cointegrating vectors, also of full column rank.

Consider now the global model, given by (3.5), which has the following error-correction form

$$\mathbf{G\Delta z}_t = \mathbf{a}_0 - (\mathbf{G} - \mathbf{H})\mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t. \tag{4.5}$$

The number of long-run relationships in the global model is similarly determined by the rank of $\mathbf{G} - \mathbf{H}$. Using (3.7) and (4.4) we first note that

$$\mathbf{G} - \mathbf{H} = \begin{pmatrix} (\mathbf{A}_0 - \mathbf{B}_0)\mathbf{W}_0 \\ (\mathbf{A}_1 - \mathbf{B}_1)\mathbf{W}_1 \\ \vdots \\ (\mathbf{A}_N - \mathbf{B}_N)\mathbf{W}_N \end{pmatrix} = \begin{pmatrix} \boldsymbol{\alpha}_0\boldsymbol{\beta}_0'\mathbf{W}_0 \\ \boldsymbol{\alpha}_1\boldsymbol{\beta}_1'\mathbf{W}_1 \\ \vdots \\ \boldsymbol{\alpha}_N\boldsymbol{\beta}_N'\mathbf{W}_N \end{pmatrix},$$

[12] A model is said to be complete if it is possible to uniquely solve for all its endogenous variables.

which can be written equivalently as

$$\mathbf{G} - \mathbf{H} = \tilde{\boldsymbol{\alpha}}\,\tilde{\boldsymbol{\beta}}',$$

where $\tilde{\boldsymbol{\alpha}}$ is the $k \times r$ block diagonal matrix of the global (structural) loading coefficients

$$\tilde{\boldsymbol{\alpha}} = \begin{pmatrix} \boldsymbol{\alpha}_0 & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\alpha}_1 & & \mathbf{0} \\ \vdots & & & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \boldsymbol{\alpha}_N \end{pmatrix}, \tag{4.6}$$

$$\tilde{\boldsymbol{\beta}} = \begin{pmatrix} \mathbf{W}_0'\boldsymbol{\beta}_0 & \mathbf{W}_1'\boldsymbol{\beta}_1 & \ldots & \mathbf{W}_N'\boldsymbol{\beta}_N \end{pmatrix}, \tag{4.7}$$

$r = \sum_{i=0}^{N} r_i$,and $k = \sum_{i=0}^{N} k_i$. It is clear that $Rank(\tilde{\boldsymbol{\alpha}}) = \sum_{i=0}^{N} Rank(\boldsymbol{\alpha}_i) = r$.

Consider now the global $k \times r$ cointegrating matrix $\tilde{\boldsymbol{\beta}}$. Each of the blocks in $\tilde{\boldsymbol{\beta}}$, namely $\mathbf{W}_i'\boldsymbol{\beta}_i$, are of dimension $k \times r_i$ with rank at most equal to r_i. Therefore, the rank of $\tilde{\boldsymbol{\beta}}$ will be at most equal to r. Namely, the number of the long-run relationships in the global model can not exceed the sum of the numbers of long-run relations that exist at the country/region specific models. Whether the number of the long-run relationships in the global model is equal to this upper bound depends on the nature of the link matrices, $\mathbf{W}_i$, and the country-specific long-run relations, $\boldsymbol{\beta}_i$.

5 Dynamic Properties, Stability Conditions and Forecasts of the GVAR Model

In this section we shall consider the dynamic properties of a slightly generalized version of the global model that allows for deterministic trends and "common global variables" such as oil prices. With this in mind first consider the following generalization of (2.1)

$$\mathbf{x}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1}t + \boldsymbol{\Phi}_i\mathbf{x}_{i,t-1} + \boldsymbol{\Lambda}_{i0}\mathbf{x}_{it}^* + \boldsymbol{\Lambda}_{i1}\mathbf{x}_{i,t-1}^* + \boldsymbol{\Psi}_{i0}\mathbf{d}_t + \boldsymbol{\Psi}_{i1}\mathbf{d}_{t-1} + \boldsymbol{\varepsilon}_{it}, \tag{5.1}$$

for $t = 1, 2, ..., T$, and $i = 0, 1, 2, ..., N$, where $\mathbf{d}_t$ is an $s \times 1$ vector of common global variables assumed to be exogenous to the global economy, and $\mathbf{a}_{i1}$ is a $k_i \times 1$ vector of linear trend coefficients. The global model associated with these country specific models is now given by

$$\mathbf{G}\mathbf{z}_t = \mathbf{a}_0 + \mathbf{a}_1\mathbf{t} + \mathbf{H}\mathbf{z}_{t-1} + \boldsymbol{\Psi}_0\mathbf{d}_t + \boldsymbol{\Psi}_1\mathbf{d}_{t-1} + \boldsymbol{\varepsilon}_t,$$

where $\mathbf{a}_0$, $\mathbf{G}$, $\mathbf{H}$ and $\boldsymbol{\varepsilon}_t$ are as already defined by (3.6) and (3.7), and

$$\mathbf{a}_1 = \begin{pmatrix} \mathbf{a}_{01} \\ \mathbf{a}_{11} \\ \vdots \\ \mathbf{a}_{N0} \end{pmatrix}, \boldsymbol{\Psi}_0 = \begin{pmatrix} \boldsymbol{\Psi}_{00} \\ \boldsymbol{\Psi}_{10} \\ \vdots \\ \boldsymbol{\Psi}_{N0} \end{pmatrix}, \ \boldsymbol{\Psi}_1 = \begin{pmatrix} \boldsymbol{\Psi}_{01} \\ \boldsymbol{\Psi}_{11} \\ \vdots \\ \boldsymbol{\Psi}_{N1} \end{pmatrix}. \tag{5.2}$$

Assuming $\mathbf{G}$ is non-singular we now have the following reduced-form global model

$$\begin{aligned} \mathbf{z}_t &= \mathbf{b}_0 + \mathbf{b}_1 t + F\mathbf{z}_{t-1} + \boldsymbol{\Upsilon}_0\mathbf{d}_t + \boldsymbol{\Upsilon}_1\mathbf{d}_{t-1} + \mathbf{u}_t, \qquad (5.3) \\ \text{for } t &= 1, 2, ..., T, T+1, ..., T+n, \end{aligned}$$

where

$$
\begin{aligned}
\mathbf{b}_i &= \mathbf{G}^{-1}\mathbf{a}_i,\ i=0,1,\ \digamma=\mathbf{G}^{-1}\mathbf{H}, \\
\boldsymbol{\Upsilon}_0 &= \mathbf{G}^{-1}\boldsymbol{\Psi}_0,\ \ \boldsymbol{\Upsilon}_1=\mathbf{G}^{-1}\boldsymbol{\Psi}_1, \text{ and } \mathbf{u}_t=\mathbf{G}^{-1}\boldsymbol{\varepsilon}_t.
\end{aligned} \tag{5.4}
$$

Suppose now that the global economy is observed over the period $t = 1,2,...,T$, and we wish to forecast $\mathbf{z}_t$ over the future periods $t = T+1, T+2, ..., T+n$, where n is the forecast horizon. We assume that the parameters and the values of the exogenous variables $\mathbf{d}_t$ for $t = T+1, T+2, ...$ are given. Then solving the difference equation (5.3) forward we obtain:

$$
\begin{aligned}
\mathbf{z}_{T+n} &= \digamma^n\mathbf{z}_T+\sum_{\tau=0}^{n-1}\digamma^\tau\left[\mathbf{b}_0+\mathbf{b}_1(T+n-\tau)\right]+ \\
&\quad \sum_{\tau=0}^{n-1}\digamma^\tau\left[\boldsymbol{\Upsilon}_0\mathbf{d}_{T+n-\tau}+\boldsymbol{\Upsilon}_1\mathbf{d}_{T+n-\tau-1}\right]+\sum_{\tau=0}^{n-1}\digamma^\tau\mathbf{u}_{T+n-\tau}.
\end{aligned} \tag{5.5}
$$

This solution has four distinct components: The first component, $\digamma^n\mathbf{z}_T$, measures the effect of initial values, $\mathbf{z}_T$, on the future state of the system. The second component captures the deterministic trends embodied in the underlying VAR model. The third component measures the effect of the global exogenous variables, $\mathbf{d}_t$, on the model's endogenous variables, $\mathbf{z}_t$. Finally, the last term in (5.5) represents the stochastic (unpredictable) component of $\mathbf{z}_{T+n}$. The point forecasts of the endogenous variables conditional on the initial state of the system and the exogenous global variables are now given by

$$
\begin{aligned}
\mathbf{z}^*_{T+n} &= E(\mathbf{z}_{T+n}\mid\mathbf{z}_T,\cup_{\tau=-1}^{n}\mathbf{d}_{T+\tau})=\digamma^n\mathbf{z}_T+\sum_{\tau=0}^{n-1}\digamma^\tau\left[\mathbf{b}_0+\mathbf{b}_1(T+n-\tau)\right]+ \\
&\quad \sum_{\tau=0}^{n-1}\digamma^\tau\left[\boldsymbol{\Upsilon}_0\mathbf{d}_{T+n-\tau}+\boldsymbol{\Upsilon}_1\mathbf{d}_{T+n-\tau-1}\right].
\end{aligned} \tag{5.6}
$$

The probability distribution function of $\mathbf{z}_{T+n}$, needed for the computation of the loss distribution of a given portfolio, can also be obtained under suitable assumptions concerning the probability distribution function of the shocks, $\boldsymbol{\varepsilon}_t$. Under the assumption that $\boldsymbol{\varepsilon}_t$ is normally distributed we have

$$
\mathbf{z}_{T+n}\mid\mathbf{z}_T,\cup_{\tau=-1}^{n}\mathbf{d}_{T+\tau}\backsim N(\mathbf{z}^*_{T+n},\boldsymbol{\Omega}_n), \tag{5.7}
$$

where $\mathbf{z}^*_{T+n}$ is given by (5.6), and

$$
\boldsymbol{\Omega}_n=\sum_{\tau=0}^{n-1}\digamma^\tau\mathbf{G}^{-1}\boldsymbol{\Sigma}\mathbf{G}'^{-1}\digamma'^\tau, \tag{5.8}
$$

where $\boldsymbol{\Sigma}$ is the $k\times k$ variance-covariance matrix of the shocks, $\boldsymbol{\varepsilon}_t$. Note that the (i,j) block of $\boldsymbol{\Sigma}$ is given by $\boldsymbol{\Sigma}_{ij}$ which is defined by (2.6). The estimation of $\boldsymbol{\Sigma}_{ij}$ and the other parameters will be addressed below.

The dynamic properties of the global model crucially depends on the eigenvalues of $\digamma$. In the trend-stationary case where all the roots of $\digamma$ lie inside the unit circle, $\mathbf{z}_{T+n}$ will have a stable distribution and will satisfy the following properties:

- The dependence of $\mathbf{z}_{T+n}$ on the initial values, $\mathbf{z}_T$, will disappear for sufficiently large values of n, the forecast horizon.

- The forecast covariance matrix, $\mathbf{\Omega}_n$, will converge to a finite value as $n \to \infty$.

- The point forecasts, $\mathbf{z}^*_{T+n}$, will exhibit the same linear trending property as the one specified in the underlying country-specific VAR models.

In contrast, when one or more roots of $\mathcal{F}$ fall on the unit circle none of the above properties hold.[13] The unit eigenvalues correspond to the unit roots and cointegrating properties of the various variables in the global VAR model.

- The multiplier matrix $\mathcal{F}^n$ converges to a non-zero matrix of fixed constants even if n is allowed to increase without bound, and the dependence of $\mathbf{z}^*_{T+n}$ on the initial values does not disappear as $n \to \infty$.

- The forecast covariance matrix, $\mathbf{\Omega}_n$, will rise linearly with n; indicating a steady deterioration in the precision with which values of $\mathbf{z}_{T+n}$ are forecast with the horizon, n.

- Finally, the linear trend in the underlying VAR model when combined with a unit root in $\mathcal{F}$ generates a quadratic trend in the level of the variables.

Some of the above undesirable features can be avoided or by passed. For example, to avoid increasing forecast error variances one could focus on forecasting growth rates (using the GVAR in levels). Quadratic trends can be eliminated by restricting the trend coefficients, $\mathbf{b}_1$ in (5.3) so that

$$\mathbf{b}_1 = (\mathbf{I}_k - \mathcal{F})\boldsymbol{\gamma},$$

where $\boldsymbol{\gamma}$ is a $k \times 1$ vector of fixed constants.[14] In terms of the trend coefficients of the underlying VAR models we have

$$\mathbf{a}_1 = (\mathbf{G} - \mathbf{H})\boldsymbol{\gamma}.$$

To impose these restrictions we need to estimate the global model (5.3), comprising all the countries/regions simultaneously - which is not feasible. An alternative procedure would be to impose the restrictions on the trend coefficients at the country/region level, namely to estimate the country/region models subject to the restrictions

$$\mathbf{a}_{i1} = (\mathbf{A}_i - \mathbf{B}_i)\boldsymbol{\kappa}_i, \tag{5.9}$$

where $\boldsymbol{\kappa}_i$ is a $(k_i + k_i^*) \times 1$ vector of fixed constants. This specification imposes $k_i + k_i^* - r_i$ restrictions on the trend coefficients, where r_i is the cointegrating rank of country-specific model, namely $Rank(\mathbf{A}_i - \mathbf{B}_i) = r_i$. This estimation problem is feasible and can be achieved by means of reduced-rank regression techniques. See Section 7 below.

[13] The case where $\mathcal{F}$ has a root outside the unit circle leads to explosive forecasts and is of little interest and could indicate model mis-specification.

[14] In the extreme case where all the roots of $\mathcal{F}$ lie on the unit circle (a case which arises if all the elements of $\mathbf{z}_t$ are independent random walk processes) then $\mathcal{F} = \mathbf{I}_k$ and $\mathbf{b}_1 = \mathbf{0}$.

6 Impulse Response Analysis

One of the important tools in the analysis of dynamic systems is the impulse response function, which characterize the possible response of the system at different future periods to the effect of shocking one of the variables in the model. For example, it may be of interest to work out the effect of a shock of a given size to the Yen/Dollar exchange rate on the evolution of real output in Germany. In carrying out such an analysis it is important that the correlation which exists across the different shocks, both within each country and across the different countries, are accounted for in an appropriate manner. In the traditional VAR literature this is accomplished by means of the orthogonalized impulse responses (OIR) a là Sims (1980), where impulse responses are computed with respect to a set of orthogonalized shocks, say $\boldsymbol{\xi}_t$, instead of the original shocks, $\boldsymbol{\varepsilon}_t$. The link between the two sets of shocks are given by

$$\boldsymbol{\xi}_t = \mathbf{P}^{-1}\boldsymbol{\varepsilon}_t,$$

where $\mathbf{P}$ is a $k \times k$ lower triangular Cheolesky factor of the variance covariance matrix, $Cov(\boldsymbol{\varepsilon}_t) = \boldsymbol{\Sigma}$, namely

$$\mathbf{P}\mathbf{P}' = \boldsymbol{\Sigma}. \tag{6.1}$$

Therefore, by construction $E(\boldsymbol{\xi}_t\boldsymbol{\xi}_t') = \mathbf{I}_k$. The $k \times 1$ vector of the orthogonalized impulse response function of a unit shock (equal to one standard error) to the jth equation on $\mathbf{z}_{t+n}$ is given by

$$\boldsymbol{\psi}_j^o(n) = \digamma^n \mathbf{G}^{-1}\mathbf{P}\mathfrak{s}_j, \; n = 0, 1, 2, ..., \tag{6.2}$$

where $\mathfrak{s}_j$ is an $k \times 1$ selection vector with unity as its jth element (corresponding to a particular shock in a particular country) and zeros elsewhere.

The orthogonalized impulse response function is usually used for small systems that admit a natural causal ordering for the variables in the VAR. But in general such a natural ordering does not exist and the OIR functions are not unique and sometime depend critically on the order in which the variables are included in the VAR. In the case of the global VAR model the orthogonalized impulse responses also depend on the order in which the variables from different regions/countries are stacked in $\mathbf{z}_t$! Mathematically, this non-invariance property of the orthogonalized impulse responses is simply due to the non-uniqueness of the Cholesky factor, $\mathbf{P}$.

An alternative approach which is invariant to the ordering of the variables and the countries in the global VAR would be to use (5.5) directly, shock only one element, say the jth shock in $\boldsymbol{\varepsilon}_t$, corresponding to the ℓth variable in the ith country, and integrate out the effects of other shocks using an assumed or the historically observed distribution of the errors. This approach is advanced in Koop, Pesaran and Potter (1996), and Pesaran and Shin (1998) and yields the generalized impulse response (GIR) function.

$$\mathbf{GI}_{z:\varepsilon_{i\ell}}(n, \; \sqrt{\sigma_{ii,\ell\ell}}, \; \mathcal{I}_{t-1}) = E\left(\mathbf{z}_{t+n} | \; \varepsilon_{i\ell t} = \sqrt{\sigma_{ii,\ell\ell}}, \; \mathcal{I}_{t-1}\right) - E\left(\mathbf{z}_{t+n} | \; \mathcal{I}_{t-1}\right), \tag{6.3}$$

where $\mathcal{I}_t = (\mathbf{z}_t, \mathbf{z}_{t-1}, ...)$ is the information set at time $t-1$, and $\mathbf{d}_t$ is assumed to be given exogenously. On the assumption that $\boldsymbol{\varepsilon}_t$ has a multivariate normal distribution and using (5.5) it is now easily seen

$$\boldsymbol{\psi}_j^g(n) = \frac{1}{\sqrt{\sigma_{ii,\ell\ell}}} \digamma^n \mathbf{G}^{-1}\boldsymbol{\Sigma}\mathfrak{s}_j, \; n = 0, 1, 2, ..., \tag{6.4}$$

which measures the effect of one standard error shock to the jth equation (corresponding to the ℓth variable in the ith country) at time t on expected values of $\mathbf{z}$ at time $t+n$. $\boldsymbol{\psi}_j^g(n)$ will be identical to $\boldsymbol{\psi}_j^o(n)$ when $\boldsymbol{\Sigma}$ is diagonal or when the focus of the analysis is on the impulse response function of shocking the first element of $\boldsymbol{\varepsilon}_t$.

6.1 Impulse Response Analysis of Shocks to the Exogenous Variables

In this sub-section we derive generalized impulse response functions for a unit shock to the $i-th$ exogenous variable, d_{it}. For this purpose we need to specify a dynamic process for the exogenous variables. Suppose $\mathbf{d}_t$ follows a first order autoregressive process:[15]

$$\mathbf{d}_t = \boldsymbol{\mu}_d + \boldsymbol{\Phi}_d \mathbf{d}_{t-1} + \boldsymbol{\eta}_t, \qquad \boldsymbol{\eta}_t \backsim i.i.d.\ (\mathbf{0}, \boldsymbol{\Sigma}_d), \tag{6.5}$$

where $\boldsymbol{\mu}_d$ is an $s \times 1$ vector of constants, $\boldsymbol{\Phi}_d$ is $s \times s$ matrix of lagged coefficients, $\boldsymbol{\eta}_t$ is an $s \times 1$ vector of shocks to the exogenous variables, and $\boldsymbol{\Sigma}_d$ is the covariance matrix of these shocks which we assume could be singular. This allows for the possibility that some of the elements of $\mathbf{d}_t$ could be perfectly predictable (such as linear trends, deterministic seasonal effects, etc.). As before the generalized impulse response function of the effect of a unit shock to the $i-th$ exogenous variable on the vector of the endogenous variables n periods ahead is defined by:

$$\mathbf{GI}_{z:d_i}(n,\ \sigma_{d,ii},\ \mathcal{I}_{t-1}) = E\left(\mathbf{z}_{t+n} \middle| \ d_{it} = \sqrt{\sigma_{d,ii}},\ \mathcal{I}_{t-1}\right) - E\left(\mathbf{z}_{t+n} \middle| \ \mathcal{I}_{t-1}\right)$$

where $\sigma_{d,ii}$ is the $i-th$ diagonal element of $\boldsymbol{\Sigma}_d$. Using (5.3) it is now easily seen that

$$\begin{aligned} \mathbf{GI}_{z:d_i}(n,\ \sigma_{d,ii},\ \mathcal{I}_{t-1}) &= \digamma\ \mathbf{GI}_{z:d_i}(n-1,\ \sigma_{d,ii},\ \mathcal{I}_{t-1}) + \boldsymbol{\Upsilon}_0 \mathbf{GI}_{d:d_i}(n,\ \sigma_{d,ii},\ \mathcal{I}_{t-1}) + \\ &\quad \boldsymbol{\Upsilon}_1 \mathbf{GI}_{d:d_i}(n-1,\ \sigma_{d,ii},\ \mathcal{I}_{t-1}), \end{aligned} \tag{6.6}$$

for $n = 0, 1, 2, ...$, where $\mathbf{GI}_{z:d_i}(n-1,\ \sigma_{d,ii},\ \mathcal{I}_{t-1}) = 0$, for $n < 1$

$$\mathbf{GI}_{z:d_i}(0,\ \sigma_{d,ii},\ \mathcal{I}_{t-1}) = \boldsymbol{\Upsilon}_0 \mathbf{GI}_{d:d_i}(0,\ \sigma_{d,ii},\ \mathcal{I}_{t-1}).$$

Similarly,

$$\mathbf{GI}_{d:d_i}(0,\ \sigma_{d,ii},\ \mathcal{I}_{t-1}) = \frac{1}{\sqrt{\sigma_{d,ii}}} \boldsymbol{\Sigma}_d \mathfrak{e}_i,$$

where $\mathfrak{e}_i$ is a $s \times 1$ selection vector with its $i-th$ element unity and other elements zero, and

$$\mathbf{GI}_{d:d_i}(n,\ \sigma_{d,ii},\ \mathcal{I}_{t-1}) = \boldsymbol{\Phi}_d \mathbf{GI}_{d:d_i}(n-1,\ \sigma_{d,ii},\ \mathcal{I}_{t-1}), \text{ for } n = 1, 2, ...$$

Hence

$$\mathbf{GI}_{d:d_i}(n,\ \sigma_{d,ii},\ \mathcal{I}_{t-1}) = \frac{1}{\sqrt{\sigma_{d,ii}}} \boldsymbol{\Phi}_d^n \boldsymbol{\Sigma}_d \mathfrak{e}_i.$$

[15] The analysis is easily extended to higher order processes.

Substituting this result in (6.6) we have

$$\mathbf{GI}_{z:d_i}(n, \sigma_{d,ii}, \mathcal{I}_{t-1}) = \varGamma\ \mathbf{GI}_{z:d_i}(n-1, \sigma_{d,ii}, \mathcal{I}_{t-1}) + \frac{1}{\sqrt{\sigma_{d,ii}}}(\mathbf{\Upsilon}_0\mathbf{\Phi}_d + \mathbf{\Upsilon}_1)\mathbf{\Phi}_d^{n-1}\mathbf{\Sigma}_d\mathfrak{e}_i, \tag{6.7}$$

for $n = 1, 2, ...$, where

$$\mathbf{GI}_{z:d_i}(0,\ \sigma_{d,ii},\ \mathcal{I}_{t-1}) = \frac{1}{\sqrt{\sigma_{d,ii}}}\mathbf{\Upsilon}_0\mathbf{\Sigma}_d\mathfrak{e}_i. \tag{6.8}$$

In the simple case where $\mathbf{d}$ is a scalar variable (such as oil prices) $\frac{1}{\sqrt{\sigma_{d,ii}}}\mathbf{\Sigma}_d\mathfrak{e}_i = \sqrt{\sigma_{d,ii}}$.

7 Estimation

As was pointed out earlier a system estimation of the VAR model in (5.3) will not be possible even for moderate values of N. The unconstrained estimation of (5.3) would involve estimating a large number of parameters often greater than the number of available observations! But the modelling approach set out above is feasible even for a relatively large of number of country/regions. This is due to the fact that the weights w_{ij}^y, w_{ij}^p, w_{ij}^q, w_{ij}^e, w_{ij}^r, and w_{ij}^n are not estimated simultaneously with the other country-specific parameters but are computed from cross-country data on trade and capital flow accounts. Also the estimation of the country-specific parameters are carried out on a country-by-country basis, rather than simultaneously.

For simulation of portfolio loss distributions (and for impulse response analysis) we also need to estimate the covariance matrix of $\boldsymbol{\varepsilon}_t$. Denote the least squares (or reduced rank regression) estimates of $\boldsymbol{\varepsilon}_{it}$ by $\hat{\boldsymbol{\varepsilon}}_{it}$, then we have

$$Co\hat{v}(\boldsymbol{\varepsilon}_{it}, \boldsymbol{\varepsilon}_{jt}) = T^{-1}\sum_{t=1}^{T}\hat{\boldsymbol{\varepsilon}}_{it}\hat{\boldsymbol{\varepsilon}}_{jt}', \tag{7.1}$$

$$Co\hat{v}(\boldsymbol{\varepsilon}_t) = \begin{pmatrix} Co\hat{v}(\boldsymbol{\varepsilon}_{0t}, \boldsymbol{\varepsilon}_{0t}) & Co\hat{v}(\boldsymbol{\varepsilon}_{0t}, \boldsymbol{\varepsilon}_{1t}) & \cdots & Co\hat{v}(\boldsymbol{\varepsilon}_{0t}, \boldsymbol{\varepsilon}_{Nt}) \\ Co\hat{v}(\boldsymbol{\varepsilon}_{1t}, \boldsymbol{\varepsilon}_{0t}) & Co\hat{v}(\boldsymbol{\varepsilon}_{1t}, \boldsymbol{\varepsilon}_{1t}) & \cdots & Co\hat{v}(\boldsymbol{\varepsilon}_{1t}, \boldsymbol{\varepsilon}_{Nt}) \\ \vdots & \vdots & & \vdots \\ Co\hat{v}(\boldsymbol{\varepsilon}_{Nt}, \boldsymbol{\varepsilon}_{0t}) & Co\hat{v}(\boldsymbol{\varepsilon}_{Nt}, \boldsymbol{\varepsilon}_{1t}) & \cdots & Co\hat{v}(\boldsymbol{\varepsilon}_{Nt}, \boldsymbol{\varepsilon}_{Nt}) \end{pmatrix}, \tag{7.2}$$

$$\begin{aligned} \hat{\boldsymbol{\varepsilon}}_{it} &= \mathbf{x}_{it} - \hat{\mathbf{a}}_{i0} - \hat{\mathbf{a}}_{i1}t - \hat{\mathbf{\Phi}}_i\mathbf{x}_{i,t-1} - \\ &\quad \hat{\mathbf{\Lambda}}_{i0}\mathbf{x}_{it}^* - \hat{\mathbf{\Lambda}}_{i1}\mathbf{x}_{i,t-1}^* - \hat{\mathbf{\Psi}}_{i0}\mathbf{d}_t - \hat{\mathbf{\Psi}}_{i1}\mathbf{d}_{t-1}. \end{aligned} \tag{7.3}$$

The estimates $\hat{\mathbf{a}}_{i0}$, $\hat{\mathbf{a}}_{i1}$, $\hat{\mathbf{\Phi}}_i$, $\hat{\mathbf{\Lambda}}_{i0}$, $\hat{\mathbf{\Lambda}}_{i1}$, $\hat{\mathbf{\Psi}}_{i0}$, and $\hat{\mathbf{\Psi}}_{i1}$ can be obtained by OLS method or by the reduced rank procedure directly applied to (5.1). The OLS estimation is clearly much simpler, but suffers from the shortcoming that it does not fully allow for the fact that all the six factors used in the model are most likely to have unit roots; nor does it take into account the important possibility that the level of domestic and foreign variables may be tied together in the long-run (the phenomenon known as cointegration in the econometric literature). To deal with the unit root problem many researchers in the past have estimated the VAR model in first-differences (using rates of changes of the factors rather than their logarithms). But the first-differencing operation can be highly inefficient when there are in fact cointegrating relations amongst the factors and can be avoided by the reduced rank regression approach. The technical details of identification and estimation of country-specific models by the reduced rank regression techniques is provided in Appendix A.

8 Cross-Country Aggregation in Global VAR Modelling

One of the strengths of the global vector autoregressive (GVAR) modelling approach set out above lies in its flexibility in taking account of the various inter-linkages in the global economy in the context of a truly multi-country setting. But it has the disadvantage that it could be computationally demanding when a large number of countries (say100 or more) are included in the model. One possible way of reducing the computational burden is to apply the approach to a few key countries (say G7) individually, and then aggregate the remaining countries into 5–10 blocks or regions. This section considers how regional models can be constructed from the underlying country-specific models.[16]

Consider a given region i (South Asia, North Africa, or the Middle East, for example) composed of N_i countries. Denote the vector of country-specific variables in region i by $\mathbf{x}_{i\ell t}$, and the associated foreign variable vector by $\mathbf{x}^*_{i\ell t}$, where $i = 0, 1, 2, ..., R$ and $\ell = 1, 2, ..., N_i$. We shall continue to assume that the reference country (or region) is denoted by 0.[17] The country-specific model for country ℓ in region i is given by

$$\mathbf{x}_{i\ell t} = \mathbf{a}_{i\ell 0} + \mathbf{a}_{i\ell 1} t + \mathbf{\Phi}_{i\ell}\mathbf{x}_{i\ell,t-1} + \mathbf{\Lambda}_{i\ell 0}\mathbf{x}^*_{i\ell t} + \mathbf{\Lambda}_{i\ell 1}\mathbf{x}^*_{i\ell,t-1} + \mathbf{\Psi}_{i\ell 0}\mathbf{d}_t + \mathbf{\Psi}_{i\ell 1}\mathbf{d}_{t-1} + \boldsymbol{\varepsilon}_{i\ell t}, \tag{8.1}$$

which is an adaptation of (2.1). The problem of aggregating the N_i countries within region i centers on the heterogeneity of the coefficient matrices $\mathbf{\Phi}_{i\ell}$, $\mathbf{\Lambda}_{i\ell 0}$, and $\mathbf{\Lambda}_{i\ell 1}$ associated with the country-specific variables. The cross-country heterogeneity of the remaining parameters does not pose any special problem. There will always be an aggregation problem so long as $\mathbf{\Phi}_{i\ell}$, $\mathbf{\Lambda}_{i\ell 0}$, and $\mathbf{\Lambda}_{i\ell 1}$ differ across the countries in the region. But in practice it is possible to reduce the size of the aggregation error by using a weighted average of the variables $\mathbf{x}_{i\ell t}$ (and hence of $\mathbf{x}^*_{i\ell t}$); with the weights reflecting the relative importance of the countries in the region. Let $w^0_{i\ell}$ be the weight of country ℓ in the region i. Clearly, $\sum_{\ell=1}^{N_i} w^0_{i\ell} = 1$. Then aggregating the countries in the region using these weights we have

$$\begin{aligned} \mathbf{x}_{it} &= \mathbf{a}_{i0} + \mathbf{a}_{i1} t + \sum_{\ell=1}^{N_i} w^0_{i\ell}\mathbf{\Phi}_{i\ell}\mathbf{x}_{i\ell,t-1} + \sum_{\ell=1}^{N_i} w^0_{i\ell}\mathbf{\Lambda}_{i\ell 0}\mathbf{x}^*_{i\ell t} + \\ & \sum_{\ell=1}^{N_i} w^0_{i\ell}\mathbf{\Lambda}_{i\ell 1}\mathbf{x}^*_{i\ell,t-1} + \mathbf{\Psi}_{i0}\mathbf{d}_t + \mathbf{\Psi}_{i1}\mathbf{d}_{t-1} + \boldsymbol{\varepsilon}_{it}, \end{aligned} \tag{8.2}$$

[16] Note that this regional aggregation is a matter of computational convenience. It is not a logical requirement of the model.

[17] A region can be a reference country if it has a unified currency. Typically $N_0 = 1$.

where

$$\mathbf{x}_{it} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 \mathbf{x}_{i\ell t},\ \mathbf{a}_{i0} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 \mathbf{a}_{i\ell 0},\ \mathbf{a}_{i1} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 \mathbf{a}_{i\ell 1}, \tag{8.3}$$

$$\mathbf{\Psi}_{i0} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 \mathbf{\Psi}_{i\ell 0},\ \mathbf{\Psi}_{i1} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 \mathbf{\Psi}_{i\ell 1},\ \boldsymbol{\varepsilon}_{it} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 \boldsymbol{\varepsilon}_{i\ell t}. \tag{8.4}$$

Using (8.2) a regional model as specified in (2.1) can be obtained. In terms of the above notations we have:

$$\begin{aligned} \mathbf{x}_{it} &= \mathbf{a}_{i0} + \mathbf{a}_{i1}t + \mathbf{\Phi}_i \mathbf{x}_{i,t-1} + \mathbf{\Lambda}_{i0}\mathbf{x}_{it}^* + \mathbf{\Lambda}_{i1}\mathbf{x}_{i,t-1}^* + \\ &\quad \mathbf{\Psi}_{i0}\mathbf{d}_t + \mathbf{\Psi}_{i1}\mathbf{d}_{t-1} + \mathbf{\Psi}_{i0}\mathbf{d}_t + \mathbf{\Psi}_{i1}\mathbf{d}_{t-1} + \boldsymbol{\xi}_{it}, \end{aligned} \tag{8.5}$$

where $\boldsymbol{\xi}_{it} = \boldsymbol{\varepsilon}_{it} + v_{it}$ is now composed of the equation errors, $\boldsymbol{\varepsilon}_{it}$, and the aggregation error defined by

$$v_{it} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 (\mathbf{\Phi}_{i\ell} - \mathbf{\Phi}_i)\mathbf{x}_{i\ell,t-1} + \sum_{\ell=1}^{N_i} w_{i\ell}^0 (\mathbf{\Lambda}_{i\ell 0} - \mathbf{\Lambda}_{i0})\mathbf{x}_{i\ell t}^* + \sum_{\ell=1}^{N_i} w_{i\ell}^0 (\mathbf{\Lambda}_{i\ell 1} - \mathbf{\Lambda}_{i1})\mathbf{x}_{i\ell,t-1}^*. \tag{8.6}$$

The region-specific foreign variables, $\mathbf{x}_{it}^*$, can be constructed either using regional trade weights or country-specific trade weights as in (2.4). In the case of the latter y_{it}^* , for example, is defined as

$$y_{it}^* = \sum_{\ell=1}^{N_i} w_{i\ell}^0 y_{i\ell t}^*,\ i = 0, 1, 2, ..., R \tag{8.7}$$

where

$$y_{i\ell t}^* = \sum_{j=0}^{R} \sum_{k=1}^{N_j} w_{i\ell,jk}^y y_{jkt},\ \ell = 1, 2, ..., N_i,\ i = 0, 1, 2, ..., R, \tag{8.8}$$

$w_{i\ell,jk}^y$ is the share of country k in region j in the total trade of country ℓ in region i.

$$N = \sum_{i=0}^{R} N_i. \tag{8.9}$$

The importance of the aggregation error depends on the extent and nature of the differences in the coefficient matrices $\mathbf{\Phi}_{i\ell}$, $\mathbf{\Lambda}_{i\ell 0}$ and $\mathbf{\Lambda}_{i\ell 1}$ across the different countries in the region. The aggregation error can be minimized by choosing regions with similar economies (as far as possible) and by a sensible choice of the weights, $w_{i\ell}^0$. Importance of countries in a region is best measured by their output levels and for comparability it is important that they are measured in purchasing power parity (PPP) dollars. The weights $w_{i\ell}^0$ can be computed using PPP-adjusted GDP series for a given year or based on averages computed over several years. It may also be desirable to update the weights on a rolling basis; say by using five-yearly lagged moving-averages.

In view of the above analysis the regional variables need to be constructed from country-specific variables using the following (logarithmic) weighted averages[18]

$$y_{it} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 y_{i\ell t}, \ \ p_{it} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 p_{i\ell t}, \ \ q_{it} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 q_{i\ell t}, \tag{8.10}$$

$$e_{it} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 e_{i\ell t}, \ \ \rho_{it} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 \rho_{i\ell t}, \ \ m_{it} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 m_{i\ell t}. \tag{8.11}$$

Notice that in constructing the regional variables y_{it}, p_{it}, e_{it}, ... from the country-specific variables $y_{i\ell t}$, $p_{i\ell t}$, $e_{i\ell t}$, ... one simply needs to use country-specific variables measured in their domestic currencies. Notice that $e_{i\ell t}$ stands for the exchange rate of country ℓ in region i, in terms of US dollars.

9 An Empirical Application

In this section we illustrate our approach by estimating a global quarterly model over the period 1979Q1-1999Q1 comprising of USA, Germany, Japan, China and 25 other countries aggregated into 5 regions: Western Europe, Central Europe, South East Asia, Middle East, and Latin America. The details of these 9 country/region classifications are given in Table 1.

Table 1

Countries/Regions in the GVAR Model

US	**Germany**	**Japan**
C. Europe	**S E Asia**	**Latin America**
·Poland	·Korea	·Argentina
·Czech Republic	·Thailand	·Brazil
·Hungary	·Indonesia	·Chile
	·Malaysia	·Peru
	·Philippines	·Mexico
	·Singapore	
W. Europe	**Middle East**	**China**
·France	·Kuwait	
·Italy	·Saudi Arabia	
·UK	·Turkey	
·Spain		
·Belgium		
·Netherlands		
·Switzerland		
·Austria		

These countries comprise about 70% of world GDP. They were chosen largely because the major banks in G-7 countries have most of their exposure in this set of countries. Noticeably absent are Scandinavian countries, Africa and

[18] The weights $w_{i\ell}^0$ could be changed at fixed time intervals, say every 5 years, in order to capture secular changes in the composition of the regional output. However, changing these weights too frequently could mask the cyclical movements of the regional output being measured.

Australia-New Zealand. Future extensions of the model will look to incorporate countries from these regions. These country groupings were then converted into regional data using GDP shares for each country in the region weighted by the GDP share. For this we used purchasing power parity (PPP)-weighted GDP figures, which is thought to be more reliable than using weights based on US dollar GDPs.[19] For modelling purposes we distinguish between the regions with developed capital markets namely USA, Germany, Japan, Western Europe, South East Asia and Latin America, and the rest. As noted earlier US dollar will be used as the numaraire and its value in terms of the other currencies will be determined outside the US model.

The first step in the global VAR modelling exercise is to construct the foreign country/region specific ("starred") variables from the domestic variables using the relations (2.4).[20] For the weights we decided to rely exclusively on trade weights based on the UN Direction of Trade Statistics. Information on capital flows were not of sufficiently high quality and tended to be rather volatile. The 9×9 matrix of the trade weights computed as shares of exports and imports over the 1996-98 period is presented in Table 2. The trade shares of each country/region is displayed by columns. This matrix plays a key role in linking up the models of the different regions together and shows the degree to which one country/region depends on the remaining countries. For example, it can be clearly seen that Latin America is much more integrated/dependent on the US economy than the rest of the regions, whilst the Middle East is more integrated with the economies of the Western Europe and Germany, and the bulk of China's trade is with USA, Japan, Germany, Western Europe, and South East Asia.

The second stage in the modelling process is to select appropriate transformations of the domestic and foreign variables for inclusion in the country/region specific cointegrating VAR models. The reduced rank regression techniques reviewed in Appendix (A) are based on the assumption that the underlying endogenous and exogenous variables to be included in the country/region specific models are approximately integrated of order unity. To ascertain the order of integration of the variables in the country/region specific variables in Tables 3a and 3b we present augmented Dickey-Fuller (ADF) statistics for the levels, first differences and the second differences of the domestic and country/region specific foreign variables. To ensure comparability all these statistics are computed over the same period, 1980Q2 to 1991Q1, using an underlying univariate autoregressive process of order 5, with a linear trend in the case of the levels and an intercept term only in the case of the first and second-differences.

Generally speaking, the result of these unit root tests are in line with what is known in the literature. Interest rates (domestic and foreign) and real equity prices (domestic and foreign) are unambiguously $I(1)$ across all countries/regions. The same also applies to exchange rates with the notable exception of Latin America. In the case of Latin America the hypothesis that exchange rate is $I(2)$ can not be rejected. Mainly as a consequence, it is also not possible to reject the hypothesis that the US-specific foreign exchange rate

[19] Information on data sources and the construction of regional data series are provided in the Appendix (B).

[20] The details and the sources of the primary macro variables are provided in Appendix (B).

variable defined by

$$e_{US}^{*}=\sum_{j=1}^{8}w_{US,j}e_{j}, \tag{9.1}$$

is an $I(2)$ variable. (see the first column of Table 3b). There are two possible ways of dealing with this problem. We could decide to model Δe instead of e, but this will most likely involve over-differencing and efficiency loss in the case of the seven remaining regional models. Another, arguably more attractive, alternative would be to include the real exchange rate $(e-p)$ in the regional models. The hypothesis that $e^{*}-p^{*}$ is $I(1)$ now prevails across all countries, and the hypothesis that $e-p$ is $I(1)$ is not supported only in the case of Latin America, although at a lower level of significance (as compared to the ADF test applied to e) and could be due to the low power of the ADF test in small samples.

As far as the order of integration of the remaining three variables are concerned, the evidence is less clear cut, which is partly due to uneven data quality across the countries and the relatively short sample period under consideration. Using the 95% significance level, a unit root in real outputs is not rejected in any of the nine regions. However, in the case of three of the regions (Japan, Latin America and China) the ADF statistics seem to suggest that real output could be $I(2)$! This is clearly implausible and again could be due to poor data quality in the case of Latin America and China. The result for Japan is, however, difficult to explain, although Japan's national income statistics are not regarded as particularly reliable. Also the $I(2)$ results for Latin America and China are on the borderline and do not hold if we adopt a 90% significance level. A similar argument also applies to foreign output variables, y^{*} and real money balances, m and m^{*}. Therefore, it seems appropriate for our purposes to treat all these variables approximately as $I(1)$. Finally, for the price variables the test results suggest that the general price level, p, is $I(1)$ in four regions and $I(2)$ in the remaining five regions. The situation is more clear cut with respect to p^{*}, which is $I(2)$ for all regions with the exception of Latin America. Therefore, within our modelling framework we ought to be using inflation rates, Δp and Δp^{*}, that are at most $I(1)$, instead of the price levels.

In view of the above results, the endogenous variables of the US model were selected to be real output (y_{US}), the rate of inflation (Δp_{US}), the level of interest rate (r_{US}), the real money balances (m_{US}), and the real equity prices (q_{US}), all measured in logarithms as defined in (2.3). Within the GVAR framework the value of the US dollar is determined outside the US model, and the US-specific real exchange rate variable, $e_{US}^{*}-p_{US}^{*}$, is then included as an I(1) exogenous (or long-run forcing) variable in the US model.[21] Given the size of the US economy and its importance for global economic interactions, no other foreign-specific exogenous variable was considered for inclusion in the US model. But to control for important global political events, the logarithm of oil prices (p^{o}) were included as an exogenous I(1) variable in *all* the country/region specific models.[22]

[21] The weights $w_{US,j}$, $j=1,2,...,8$ are given in the first column of Table 2.

[22] The ADF statistics computed over the period 1980Q2-1999Q1 for the level and first-differences of oil prices were -2.27 and -4.74, respectively; thus providing empirical support for treating oil prices as an $I(1)$ variable.

In the case of Japan, Western Europe, Germany, South East Asia and Latin America with advanced capital markets we chose $(y_j, \Delta p_j, r_j, e_j - p_j, m_j, q_j)$ as their endogenous variables, and $\left(y_j^*, \Delta p_j^*, r_j^*, m_j^*, q_j^*, p^o\right)$ as their exogenous variables. Notice that e_j^* is excluded from the set of exogenous variables on the grounds of its close relationship to e_j.[23] For the remaining regions (Central Europe, Middle East and China) the set of included endogenous and exogenous variables were $(y_j, \Delta p_j, r_j, e_j - p_j, m_j)$ and $\left(y_j^*, \Delta p_j^*, r_j^*, m_j^*, q_j^*, p^o\right)$, respectively.

The next step in the analysis is to estimate region-specific cointegrating VAR models and identify the rank of their cointegrating space. The order of the underlying VAR models was taken to be 1. This choice was dictated to us by the small number of time series observations that were available to us relative to the number of unknown parameters in each of the regional models. The "trace" and "maximum eigenvalue" test statistics for each of the nine regions together with the associated 90% and 95% critical values are summarized in Tables 4a-4c.[24] It is known that both of these statistics tend to over-reject in small samples, with the extent of over-rejection being much more serious for the maximum eigenvalue as compared to the trace test. Using Monte Carlo experiments it is also shown that the maximum eigenvalue test is generally less robust to departures from normal errors than the trace test.[25] The latter point is particularly relevant to our applications since they contain equity prices, exchange rates and interest rates, all of which exhibit significant degrees of departures from normality. We shall therefore base our analysis on the trace statistics. Accordingly, for Western Europe we obtain 5 cointegrating relations, for Central Europe, Germany and Japan 4 each, for Latin America, Middle East, China and South East Asia 3 each, and only one cointegrating relation for the US. Therefore, the total number of cointegrating relations in the global model can at most be equal to $r = \sum_{i=0}^{N} r_i = 30$, where $N = 8$.[26]

The global model can be obtained by combining the region-specific models as in Section 3. The long-run and short-run dynamic properties of the global model are determined by the global cointegrating matrix, $\tilde{\boldsymbol{\beta}}$, given by (4.7), and the eigenvalues of $\mathcal{F} = \mathbf{G}^{-1}\mathbf{H}$, defined by (5.4). Since the global model contains 50 endogenous variables and the rank of $\tilde{\boldsymbol{\beta}}$ is at most 30, it then follows that $\mathcal{F}$ must have at most 20 eigenvalues that fall on the unit circle.[27] It is encouraging that our application does in fact satisfy this property. The matrix $\mathcal{F}$, estimated from the region-specific models has exactly 20 eigenvalues that fall on the unit circle with the remaining 30 eigenvalues having moduli all less than unity.[28] Amongst the latter set, the 3 largest eigenvalues (in moduli) are 0.9397, 0.8654, and 0.8467, thus ensuring a reasonably fast rate of convergence of the model to

[23] See Section 2 for a more detailed discusion.

[24] These statistics are computed using VAR(1) specifications with restricted trends, except for the US where we estimated a VAR(1) model with unrestricted intercepts. Inclusion of a linear trend in the US model resulted in unstable roots for the global model.

[25] See, for example, Cheung and Lai (1993).

[26] See Section 4 for further details.

[27] Notice that $rank(\tilde{\boldsymbol{\beta}}) = rank(\mathbf{G} - \mathbf{H})$, and for a non-singular matrix $\mathbf{G}$, then $rank(\mathbf{I} - \mathcal{F}) = rank(\tilde{\boldsymbol{\beta}})$.

[28] Out of these 30 eigenvalues, 22 (11 pairs) were complex, that produce the damped cyclical character of the generalized impulse response functions discussed below. The eigenvalues with the three largest complex part were $0.2388 \pm 0.3183i$, $0.6888 \pm 0.2339i$, $0.7190 \pm 0.1657i$, where $i = \sqrt{-1}$.

its steady state once shocked. These results also establish that the (implied) global model also forms a cointegrating system with 30 long-run relations and a stable error-correcting representation.[29] In particular, the effects of shocks on the long-run relations of the global economy will eventually disappear. The decay rate is bounded by 0.9397. However, due to the unit root properties of the global model (as characterized by the unit eigenvalues of F) global or regional shocks will have permanent effects on the levels of the variables such as real outputs, interest rates or real equity prices.

9.1 An In-sample Root Mean Square Forecast Error Comparison

Empirical evaluations of the global model can be carried out at two levels, namely at the level of the regional models, each treated separately conditional on the foreign-specific variables; and at the global level using the reduced-form specification (5.3). The former exercise is informative regarding the precision with which the parameters of the different regional models are estimated. While the latter can be used to evaluate the in-sample fit of the global model as compared to alternative benchmarks. The in-sample fit of the individual regional models are likely to be exaggerated as they are conditional on contemporaneous variables. In contrast the in-sample fit of the global model only depends on contemporaneous changes in oil prices and can be more indicative of the potential use of the model for forecasting, impulse response analysis and in risk management.

The one-step ahead in-sample root mean square forecast errors (RMSFE's) of the global model computed over the period 1979Q3 to 1999Q1, grouped by factors and regions are given in Table 5. As benchmarks we also computed RMSFEs based on the following random walk models with drifts:[30]

$$
\begin{aligned}
y_{it} &= y_{i,t-1} + \mu_i^y + \varepsilon_{it}^y, \ \Delta p_{it} = \Delta p_{i,t-1} + \mu_i^p + \varepsilon_{it}^p, \\
q_{it} &= q_{i,t-1} + \mu_i^q + \varepsilon_{it}^q, \ e_{it} - p_{it} = e_{it} - p_{i,t-1} + \mu_i^e + \varepsilon_{it}^e, \\
r_{it} &= r_{i,t-1} + \mu_i^r + \varepsilon_{it}^r, \ m_{it} = m_{i,t-1} + \mu_i^m + \varepsilon_{it}^m.
\end{aligned}
$$

The RMSFEs associated with these benchmark models are also summarized in Table 5. As can be seen the RMSFEs generated by the GVAR model are all smaller than those of the benchmark model except for real equity prices in the case of Western Europe and Germany, and the interest rate for the US. At the global level, using averages of the RMSFEs across the regions, the GVAR model performs better than the benchmark model in the case of all factors, although the degree of its out-performance differs markedly depending on the factor being considered. For forecasting real output, inflation, and real money balances, the GVAR model outperforms the benchmark model by 31.8%, 18.9%, and 18.1%, respectively. But for forecasting equity prices and interest rates the

[29] In constructing the global model only exact identifying restrictions are imposed on the cointegrating relations of the underlying regional models. But in principle further long-run (or even short run) over-identifying restrictions could also be imposed, as in Garratt et al. (2001). However, this will require detailed long-run structural analysis of the individual regions and will be beyond the scope of the present paper.

[30] The drift parameters are estimated over the same sample period, namely 1979Q3 to 1999Q1.

extent of outperformance are very small, around 4.5% and 3.4%, respectively. For real exchange rate the results are somewhere in between, namely 15.4%. However, these estimates should be viewed as indicative rather than definite, since they do not take account of parameter uncertainty and some will not be statistically significant. These results also suggest that it may be agood idea to consider restricted versions of the GVAR, say by imposition of over-identifying restrictions on the long-run parameters and/or the imposition of simplifying restrictions on the short-run coefficients. However, these and other extensions of the GVAR model lie outside the scope of the present paper, but are clearly worth pursuing.

9.2 Generalized Impulse Response Functions for Selected Shock Scenarios

The time-profiles of the effects of a variety of shocks of interest on the global economy can now be computed by means of the generalized impulse response function (GIRF) discussed in Section 6. Note that the ordering of the variables does not matter for our approach.[31] There are many shock scenarios of interest that could be investigated. Here we consider the following ones:

- A one standard error negative shock (a negative "unit" shock) to US equity prices.
- A one standard error positive shock to US interest rates.
- A one standard error negative shock to equity markets in South East Asia.

We could examine the time profiles of the effects of these shocks either on the endogenous variables of a particular region, or on a given variable across all the regions.

9.2.1 A Negative Shock to US Equity Prices

Figure 1 displays the impacts of shocks to US equity market on equity prices worldwide. On impact, the fall in the US equity prices causes prices in all equity markets to fall as well but by smaller amounts: 3.6% in Western Europe, 5.8% in Germany, 2.4% in Japan, 3.0% in South East Asia, and 5.5% in Latin America, as compared to a fall of 6.5% in the US. (See Table 6). However, over time the fall in equity prices across the regions start to catch up with the US and gets significantly amplified in the case of the two emerging markets, Latin America and the Far East Asia. In the case of these markets the fall in equity prices reaches 13.6% and 14.3%, respectively. These estimates should, however, be viewed with caution. They are likely to be poorly estimated with large standard errors, particularly those that refer to long forecast horizons.[32] Nevertheless, the relative position and pattern of the impulse response functions could still be quite informative. For example, they confirm the pivotal role played by the

[31] For a detailed discussion, see Section 6.

[32] It is possible to compute standard errors for the generalized impulse responses using bootstrap techniques. See, for example, Garratt et al. (2001). But this would be a highly computer intensive exercise and it is not clear to us that it will add much to our overall conclusions.

US stock market in the global economy, and suggest that in the longer run the emerging markets are likely to be more seriously affected than the markets in advanced economies when the US equity market is shocked. See Figure 1.

The time profiles of the effects of the shock to the US equity market on real output across the different regions are shown in Figure 2. The second panel of Table 6 provides the associated point estimates for a number of selected horizons. The impact effects of the fall in US equity market on real output are negative for most regions, but rather small in magnitude. After one year real output shows a fall of around -0.30% in Western Europe and Germany, -0.37% in the US, -0.34% in Latin America, and -0.23% in South East Asia. Japanese output only begins to be negatively affected by the adverse US stock market shock much later. The three regions without capital markets are either not affected by the shock or even show a rise in output (in particular the Central European region). Once again these point estimates should be treated with caution.[33]

9.2.2 A Positive Shock to US Interest Rates

The effects of a one standard error rise in the level of US interest rates on real equity prices and real output across the different regions are summarized in Table 7 and displayed in Figures 3 and 4.[34] Table 7 also provides the point estimates of the effects of the interest rate shock on inflation, interest rates and real exchange rates for selected horizons. The important role played by US interest rates in the global economy can be clearly seen from these results. On impact the increase in US interest rates causes the equity prices to decline in all markets, with the decline being most pronounced in Latin America. It is also interesting that the long-run impacts of the interest rate rise are felt most on equity prices in the emerging markets, with the Western Europe following closely behind. Once again the US stock market seems to have been relatively robust to the adverse move in interest rates.

The output effects of the interest rate rise are mixed. Notably on impact they are adverse in the case of all regions except for South East Asia, Latin America and the Middle East. However, these effects are rather small and could be statistically insignificant. Over the long-run, however, the effects of the shock on real outputs are negative in the case of 6 out of the 9 regions; the notable exceptions being US, Germany and Central Europe. The emerging markets are the ones that are most affected by the adverse interest rate shock: -1.15% in the case of Latin America and -0.79% in the case of South East Asia after 5 years.

The effects of the interest rate rise on the rate inflation across the different regions is mixed, and tend to switch signs as one moves from the impact effects to the long-run effects. On impact the effects of the rise in the US interest on inflation are negative only in the case of Japan (-0.04), Latin America (-0.28) and Central Europe (-0.44). However, in the longer run the rise in interest

[33]Table 6 also provides point estimates of the time profiles of the effects of the adverse US stock market shock on inflation, interest rates and real exchange rates. Overall the pattern of the impulse responses across the regions seem plausible, although space does not permit a detailed discussion of these results here.

[34]A one standard error change in the US interest rate is approximately equivalent to a 28 basis points change in the interest rate at a quarterly rate, or around 1.12% on an annual basis.

rate seems to have had the desired effect of reducing the rate of inflation in all regions except for China, Eastern Europe, and ironically the US, although the magnitude of the long-run effect on the US inflation is very small (0.09) and most likely would not be statistically significant. The results for China and Eastern Europe should be viewed with special caution due to poor data quality and the centrally planned nature of these economies for the large part of the sample period under consideration.

Finally, it is interesting that, as predicted by the uncovered interest parity hypothesis, in the long run the rise in the US interest has been associated with a rise in the interest rates across all the regions. See the fourth panel of Table 7.

9.2.3 A Negative Shock to Equity Markets in South East Asia

Given the interest in the effects of the 1997 South Asian Crisis and its possible contagion effects, here we consider the generalized impulse response functions for a one standard error negative shock to equity prices in South East Asia (SEA).[35] The one standard error shock is equivalent to 8.3% decline in SEA's equity prices and on impact has small positive effects on Japan's and US equity prices (1.25% and 0.28%, respectively) and relatively small negative effects on equity prices in Western Europe, Germany and Latin America (-0.12%, -1.85% and -0.57%, respectively). See the first panel of Table 8 and Figure 5. But over time these effects accumulate and after two years all markets are adversely affected with the exception of the US. The US equity market seems to have been reasonably robust to the South Asian Crisis. It is also interesting to note that in the longer run the Western Europe and Germany seem to be more vulnerable to the South Asian Crisis than Japan.

As to be expected the ouput effects of the negative shock to the SEA's equity markets is much more muted when compared to its effects on equity prices. On impact real output declines noticeably only in the case of China (-0.65%). See the second panel of Table 8 and Figure 6. Even after one year adverse effects of the shock can be seen only in the case of real output in Western Europe (-0.08%), Germany (-0.17%), South East Asia (-0.60%) and China (-0.18%). Once again the impulse responses suggest that Japan, US and Latin America are likely to be reasonably robust to adverse shocks to the South East Asian equity markets. At first this result seems rather surprising considering the relatively strong trade links that exists between South East Asia, Japan and US. (see Table 2). However, it largely reflects the apparently weak links that exists between the equity markets of these economies as can be seen from the first panel of Table 8 and discussed above. The impulse responses of the effects of the negative shock to SEA's equity markets on inflation, interest rates and exchange rates are summarized in the bottom three panels of Table 8. Other implications of the South Asian Crisis (such as an adverse shock to exchange rates) can also be investigated using the global VAR modelling tools developed in this paper.

[35] Our framework can also be used to investigate the contagion effects within the South East Asian region. However, this would have required a much higher degree of regional disaggregation and could be the subject of a separate study.

10 Concluding Remarks

In this paper, we develop an operational framework for global macroeconomic modeling, building on recent advances in the literature on the analysis of cointegrating systems. We demonstrate the feasibility of this approach by linking up nine separate vector error-correcting regional models estimated using quarterly observations over the period 1979Q1-1999Q1. The resultant global model is shown also to be error-correcting with damped cyclical properties. Using generalized impulse response analysis, we examine the propagation of shocks across factors and regions.

The focus of the model is very much on constructing a compact, cohesive and coherent representation of factor and regional interdependencies. The model allows for interaction amongst the different economies through three separate but interrelated channels:

1. Direct dependence of the relevant macro-factors on their country-specific foreign counterparts and their lagged values;

2. Dependence of the country-specific variables on common global exogenous variables such as oil prices and possibly other variables controlling for major global political events;

3. Non-zero contemporaneous dependence of shocks in country i on the shocks in country j, measured via the cross-country covariances.

Thus, for instance, we are able to account for inter-linkages between equity market movements in South East Asia and output in Germany. While accurate point forecasts are not the primary goal of the model, it does compare favorably to a chosen benchmark model, a random walk with drift, where one-step-ahead in-sample root mean square forecast errors (RMSFE) is the evaluation metric. Of 50 variables modeled across a total of nine regions, the GVAR model performs better than the benchmark in all but three cases. It does relatively better forecasting real output, inflation, real money and exchange rates than it does equity prices and interest rates.[36]

The original motivation for developing this model was as a risk management tool for commercial, and perhaps even central, banks. By engaging in commercial lending to companies whose fortunes fluctuate with aggregate demand, a bank is ultimately exposed to macroeconomic fluctuations. This can be mitigated through international diversification. However, precisely because economic fluctuations are correlated across factors and countries, it fosters the need for a compact global macroeconometric model which explicitly allows for such interconnections and interdependencies. We plan to address the issue of linking the loss distribution of a bank's credit asset portfolio to the macroeconomic fundamentals in a companion paper (Pesaran, Schuermann, Treutler and Weiner, 2001).

Because of the focus on modeling interlinkages, the model can readily be used to shed light on the analysis of a variety of transmission mechanisms, contagion effects, and testing of long-run theories (for instance, purchasing power parity) in a global setting. We can think of several other applications:

- "New economic geography": a literature which sets the stage for explicitly incorporating geography into the models of economic activity through

[36] Although, we do appreciate that the RMSFE differential of the two sets of forecasts may not be statistically significant for some of the variables considered.

either domestic or international trade (see Krugman, 1993, for an introduction to the topic, and Fujita, Krugman and Venables, 1999, for a more formal treatment)

- Regional and urban economics: models of inter-regional linkages, either through city-suburb economic ties (Voith, 1998) or linkages between cities as in the "systems of cities" literature (Henderson, 1988)
- Labor mobility: consider a longer horizon, lower frequency issue of labor mobility responding to regional economic shocks; for instance, auto workers migrating from Michigan to Texas in response to oil-price shocks in the early 1980s (Blanchard and Katz, 1992).

This list is by no means exhaustive and is designed to stimulate interest, and research, of applying the GVAR framework to problems of modeling economic inter-linkages.

A Estimation of Country-Specific Models by the Reduced Rank Regression Techniques

The reduced rank estimation procedure in the case where all the variables in the model are treated as endogenous has been developed by Johansen (1988, 1995). But in the context of the GVAR model (2.1) the foreign variables, $\mathbf{x}_{it}^*$, are exogenous, and Johansen's approach needs to be modified to take this into account. Appropriate methods for estimating reduced rank regressions containing exogenous regressors have been developed by Harbo, Johansen, Nielsen and Rahbek (1998), and Pesaran, Shin and Smith (2000).

To estimate the country-specific models subject to reduced rank restrictions we first rewrite (5.1) in the error-correction form

$$\begin{aligned}\Delta\mathbf{x}_{it} &= \mathbf{a}_{i0}+\mathbf{a}_{i1}t-(\mathbf{I}_{k_i}-\mathbf{\Phi}_i)\mathbf{x}_{i,t-1}+(\mathbf{\Lambda}_{i0}+\mathbf{\Lambda}_{i1})\mathbf{x}_{i,t-1}^* \\ &\quad +(\mathbf{\Psi}_{i0}+\mathbf{\Psi}_{i1})\mathbf{d}_{t-1}+\mathbf{\Lambda}_{i0}\Delta\mathbf{x}_{it}^*+\mathbf{\Psi}_{i0}\Delta\mathbf{d}_t+\varepsilon_{it},\end{aligned}$$

or using (3.1)

$$\Delta\mathbf{x}_{it}=\mathbf{a}_{i0}+\mathbf{a}_{i1}t-(\mathbf{A}_i-\mathbf{B}_i)\mathbf{z}_{i,t-1}+\mathbf{C}_i\mathbf{d}_{t-1}+\mathbf{\Lambda}_{i0}\Delta\mathbf{x}_{it}^*+\mathbf{\Psi}_{i0}\Delta\mathbf{d}_t+\varepsilon_{it}, \quad (A.1)$$

where $\mathbf{C}_i=\mathbf{\Psi}_{i0}+\mathbf{\Psi}_{i1}$, $\mathbf{z}_{it}=(\mathbf{x}_{it}',\mathbf{x}_{it}^{*\prime})'$,and $\mathbf{A}_i$ and $\mathbf{B}_i$ are defined by (3.3). To avoid the problem of introducing quadratic trends in the level of the variables when $(\mathbf{A}_i-\mathbf{B}_i)$ is rank deficient we also impose the restrictions (5.9), namely $\mathbf{a}_{i1}=(\mathbf{A}_i-\mathbf{B}_i)\boldsymbol{\kappa}_i$. Under these restrictions (A.1) becomes

$$\Delta\mathbf{x}_{it}=\mathbf{c}_{i0}-\mathbf{\Pi}_i\mathbf{v}_{i,t-1}+\mathbf{\Lambda}_{i0}\Delta\mathbf{x}_{it}^*+\mathbf{\Psi}_{i0}\Delta\mathbf{d}_t+\varepsilon_{it}, \quad (A.2)$$

where

$$\mathbf{c}_{i0}=\mathbf{a}_{i0}+(\mathbf{A}_i-\mathbf{B}_i)\boldsymbol{\kappa}_i, \quad (A.3)$$

$$\mathbf{\Pi}_i=(\mathbf{A}_i-\mathbf{B}_i,-\mathbf{C}_i,-(\mathbf{A}_i-\mathbf{B}_i)\boldsymbol{\kappa}_i), \quad (A.4)$$

$$\mathbf{v}_{i,t-1}=\begin{pmatrix}\mathbf{z}_{i,t-1}\\ \mathbf{d}_{t-1}\\ t-1\end{pmatrix}. \quad (A.5)$$

$\mathbf{\Pi}_i$ is a $k_i\times(k_i+k^*+s+1)$ matrix and provides information on the long-run level relationships that may exist amongst the variables of the model. In the case where *all* the variables, $\mathbf{z}_{it}$ and $\mathbf{d}_t$, are $I(1)$ and are not cointegrated then $\mathbf{\Pi}_i$ will be equal to zero and (A.2) reduces to the first differenced model[37]

$$\Delta\mathbf{x}_{it}=\mathbf{a}_{i0}+\mathbf{\Lambda}_{i0}\Delta\mathbf{x}_{it}^*+\mathbf{\Psi}_{i0}\Delta\mathbf{d}_t+\varepsilon_{it}. \quad (A.6)$$

It is interesting to note that this specification leads to random walk models (augmented by oil price changes) for the global variables, $\mathbf{z}_t$. Using the solution technique of Section 3, we have

$$\mathbf{G}\boldsymbol{\Delta}\mathbf{z}_t=\mathbf{a}_0+\mathbf{\Psi}_0\boldsymbol{\Delta}\mathbf{d}_t+\varepsilon_t,$$

or

$$\boldsymbol{\Delta}\mathbf{z}_t=\mathbf{G}^{-1}\mathbf{a}_0+\mathbf{G}^{-1}\mathbf{\Psi}_0\boldsymbol{\Delta}\mathbf{d}_t+\mathbf{G}^{-1}\varepsilon_t,$$

[37] A variable is said to be $I(1)$, integrated of order 1, if it *must* be differenced *exactly* once before it becomes stationary, or $I(0)$.

where $\mathbf{G}$ and $\mathbf{\Psi}_0$ are defined by (3.7) and (5.2), respectively. Therefore, as anticipated by the analysis of Section 4, there will be no long-run relationship in the global model if there are no long-run relations in the underlying regional models.

But in general due to long-term inter-linkages that exist between domestic and foreign variables as well as between the domestic variables themselves (eg the money demand equation that relates $m_{it} - p_{it}$ to ρ_{it} and y_{it}) one would expect $\mathbf{\Pi}_i$ to be non-zero but rank deficient. The rank of $\mathbf{\Pi}_i$ identifies the number of long-run or cointegrating relationships. Rank deficiency arises when $Rank(\mathbf{\Pi}_i) = r_i$ and $r_i < k_i$. This could, for example, be due to the long-run connections that exist between domestic and foreign interest rates, prices and exchange rates (through the PPP relationship). In the more general case where $\mathbf{\Pi}_i$ is non-zero but could (possibly) be rank deficient the error-correction form of the country-specific model (A.2) needs to be estimated subject to the reduced rank restriction:

$$H_{r_i}: \qquad Rank(\mathbf{\Pi}_i) = r_i < k_i. \tag{A.7}$$

Under the assumption that $Rank(\mathbf{\Pi}_i) = r_i$ one can write

$$\mathbf{\Pi}_i = \boldsymbol{\alpha}_i \boldsymbol{\beta}_i', \tag{A.8}$$

where $\boldsymbol{\alpha}_i$ is a $k_i \times r_i$ matrix of rank r_i and $\boldsymbol{\beta}_i$ is a $(k_i + k^* + s + 1) \times r_i$ matrix of rank r_i. Using (A.8) in (A.2) we have

$$\Delta \mathbf{x}_{it} = \mathbf{c}_{i0} - \boldsymbol{\alpha}_i(\boldsymbol{\beta}_i' \mathbf{v}_{i,t-1}) + \mathbf{\Lambda}_{i0} \Delta \mathbf{x}_{it}^* + \mathbf{\Psi}_{i0} \Delta \mathbf{d}_t + \boldsymbol{\varepsilon}_{it}, \tag{A.9}$$

where $\boldsymbol{\beta}_i' \mathbf{v}_{it}$ $(= \boldsymbol{\eta}_{it})$ is an $r_i \times 1$ vector of long run or cointegrating relations, also known as error-correction terms. For a given value of $\boldsymbol{\beta}_i$ the remaining parameters, namely $\mathbf{c}_{i0}$, $\boldsymbol{\alpha}_i$, $\mathbf{\Lambda}_{i0}$, $\mathbf{\Psi}_{i0}$, and $\mathbf{\Sigma}_{ii} = cov(\boldsymbol{\varepsilon}_{it})$, can be estimated by least squares regressions of $\Delta \mathbf{x}_{it}$ on $\boldsymbol{\eta}_{it}$, $\Delta \mathbf{x}_{it}^*$, and $\Delta \mathbf{d}_t$, including only intercept terms. Recall that the trend terms are already included in $\boldsymbol{\eta}_{it}$. To estimate $\boldsymbol{\beta}_i$ we need first to identify its rank, namely the value of r_i. For this purpose we can use Johansen's approach suitably modified to deal with the present application where the model contains exogenous $I(1)$ regressors.

Suppose T observations are available, with $t = 1, 2, ..., T$. Stacking the observations on (A.9) we have:

$$\Delta \mathbf{X}_i = \mathbf{c}_{i0} \boldsymbol{\iota}_T' - \boldsymbol{\alpha}_i \boldsymbol{\beta}_i' \mathbf{V}_{i,-1} + \mathbf{\Lambda}_{i0} \Delta \mathbf{X}_i^* + \mathbf{\Psi}_{i0} \Delta \mathbf{D} + \mathbf{E}_i, \tag{A.10}$$

where $\Delta \mathbf{X}_i \equiv (\Delta \mathbf{x}_{i1}, ..., \Delta \mathbf{x}_{iT})$, $\boldsymbol{\iota}_T$ is a T-vector of ones, $\mathbf{V}_{i,-1} \equiv (\mathbf{v}_{i0}, ..., \mathbf{v}_{i,T-1})$, $\Delta \mathbf{X}_i^* = (\Delta \mathbf{x}_{i1}^*, ..., \Delta \mathbf{x}_{iT}^*)$, $\mathbf{D} \equiv (\mathbf{d}_1, ..., \mathbf{d}_T)$ and $\mathbf{E}_i \equiv (\boldsymbol{\varepsilon}_{i1}, ..., \boldsymbol{\varepsilon}_{iT})$. The log-likelihood function of the model is given by

$$\ell_T(\boldsymbol{\psi}_i; r_i) \propto -\frac{T}{2} \ln |\mathbf{\Sigma}_{ii}| - \frac{1}{2} Trace(\mathbf{\Sigma}_{ii}^{-1} \mathbf{E}_i \mathbf{E}_i'), \tag{A.11}$$

where the parameter vector $\boldsymbol{\psi}_i$ collects together the unknown parameters in $\mathbf{c}_{i0}$, $\boldsymbol{\alpha}_i \boldsymbol{\beta}_i'$, $\mathbf{\Lambda}_{i0}$, $\mathbf{\Psi}_{i0}$, and $\mathbf{\Sigma}_{ii}$. For a given value of $\boldsymbol{\beta}_i$ the estimates of $\mathbf{\Sigma}_{ii}$, $\mathbf{c}_{i0}$, $\mathbf{\Lambda}_{i0}$, $\mathbf{\Psi}_{i0}$ and $\boldsymbol{\alpha}_i$ are given by

$$\hat{\mathbf{\Sigma}}_{ii} = T^{-1} \hat{\mathbf{E}}_i \hat{\mathbf{E}}_i', \tag{A.12}$$

$$\hat{\mathbf{E}}_i = \Delta \mathbf{X}_i - \hat{\mathbf{c}}_{i0} \boldsymbol{\iota}_T' + \hat{\boldsymbol{\alpha}}_i \boldsymbol{\beta}_i' \mathbf{V}_{i,-1} - \hat{\mathbf{\Lambda}}_{i0} \Delta \mathbf{X}_i^* - \hat{\mathbf{\Psi}}_{i0} \Delta \mathbf{D}, \tag{A.13}$$

$$\hat{\mathbf{c}}_{i0} = \frac{1}{T}\left(\Delta\mathbf{X}_i + \hat{\boldsymbol{\alpha}}_i\boldsymbol{\beta}_i'\mathbf{V}_{i,-1} - \hat{\boldsymbol{\Lambda}}_{i0}\Delta\mathbf{X}_i^* - \hat{\boldsymbol{\Psi}}_{i0}\Delta\mathbf{D}\right)\boldsymbol{\iota}_T, \tag{A.14}$$

$$\hat{\boldsymbol{\alpha}}_i = \Delta\mathbf{X}_i\mathbf{V}_{i,-1}'\boldsymbol{\beta}_i\left(\boldsymbol{\beta}_i'\mathbf{V}_{i,-1}\mathbf{V}_{i,-1}'\boldsymbol{\beta}_i\right)^{-1}, \tag{A.15}$$

$$\begin{pmatrix} \hat{\boldsymbol{\Lambda}}_{i0} \\ \hat{\boldsymbol{\Psi}}_{i0} \end{pmatrix} = (\Delta\mathbf{X}_i + \hat{\boldsymbol{\alpha}}_i\boldsymbol{\beta}_i'\mathbf{V}_{i,-1})\bar{\mathbf{P}}_{\iota}\mathbf{H}_i'\left(\mathbf{H}_i\bar{\mathbf{P}}_{\iota}\mathbf{H}_i'\right)^{-1}, \tag{A.16}$$

where $\bar{\mathbf{P}}_{\iota} \equiv \mathbf{I}_T - \boldsymbol{\iota}_T(\boldsymbol{\iota}_T'\boldsymbol{\iota}_T)^{-1}\boldsymbol{\iota}_T'$, and $\mathbf{H}_i = (\Delta\mathbf{X}_i'^*, \Delta\mathbf{D}')'$. Using these estimates in (A.11) yields the concentrated log-likelihood function

$$\ell_T^c(\boldsymbol{\beta}_i; r_i) \propto \frac{T}{2}\ln\left|T^{-1}\Delta\tilde{\mathbf{X}}_i\left(\mathbf{I}_T - \tilde{\mathbf{V}}_{i,-1}'\boldsymbol{\beta}_i\left(\boldsymbol{\beta}_i'\tilde{\mathbf{V}}_{i,-1}\tilde{\mathbf{V}}_{i,-1}'\boldsymbol{\beta}_i\right)^{-1}\boldsymbol{\beta}_i'\tilde{\mathbf{V}}_{i,-1}\right)\Delta\tilde{\mathbf{X}}_i'\right|, \tag{A.17}$$

where $\Delta\tilde{\mathbf{X}}_i$ and $\tilde{\mathbf{V}}_{i,-1}$ are respectively the OLS residuals from regressions of $\Delta\mathbf{X}_i$ and $\mathbf{V}_{i,-1}$ on $(\boldsymbol{\iota}_T, \mathbf{H}_i')$. Defining the sample moment matrices

$$\mathbf{S}_{i,xx} \equiv T^{-1}\Delta\tilde{\mathbf{X}}_i\Delta\tilde{\mathbf{X}}_i',\ \mathbf{S}_{i,xv} \equiv T^{-1}\Delta\tilde{\mathbf{X}}_i\tilde{\mathbf{V}}_{i,-1}',\ \mathbf{S}_{i,vv} \equiv T^{-1}\tilde{\mathbf{V}}_{i,-1}\tilde{\mathbf{V}}_{i,-1}', \tag{A.18}$$

the maximization of the concentrated log-likelihood function $\ell_T^c(\boldsymbol{\beta}_i; r_i)$ of (A.17) reduces to the minimization of

$$\left|\mathbf{S}_{i,xx} - \mathbf{S}_{i,xv}\boldsymbol{\beta}_i\left(\boldsymbol{\beta}_i'\mathbf{S}_{i,vv}\boldsymbol{\beta}_i\right)^{-1}\boldsymbol{\beta}_i'\mathbf{S}_{i,vx}\right| = \frac{|\mathbf{S}_{i,xx}|\left|\boldsymbol{\beta}_i'\left(\mathbf{S}_{i,vv} - \mathbf{S}_{i,vx}\mathbf{S}_{i,xx}^{-1}\mathbf{S}_{i,xv}\right)\boldsymbol{\beta}_i\right|}{\left|\boldsymbol{\beta}_i'\mathbf{S}_{i,vv}\boldsymbol{\beta}_i\right|},$$

with respect to $\boldsymbol{\beta}_i$. The solution $\hat{\boldsymbol{\beta}}_i$ to this minimization problem, that is, the maximum likelihood (ML) estimator for $\boldsymbol{\beta}_i$, is given by the eigenvectors corresponding to the r largest eigenvalues $\hat{\lambda}_{i1} > ... > \hat{\lambda}_{ir_i} > 0$ of

$$\left|\hat{\lambda}_i\mathbf{S}_{i,vv} - \mathbf{S}_{i,vx}\mathbf{S}_{i,xx}^{-1}\mathbf{S}_{i,xv}\right| = 0; \tag{A.19}$$

The ML estimator $\hat{\boldsymbol{\beta}}_i$ is identified up to post-multiplication by an $r_i \times r_i$ non-singular matrix; that is, r_i^2 just-identifying restrictions on $\boldsymbol{\beta}_i$ are required for exact identification.[38] The resultant maximized concentrated log-likelihood function $\ell_T^c(\boldsymbol{\beta}_i; r_i)$ at $\hat{\boldsymbol{\beta}}_i$ of (A.17) is

$$\ell_T^c(r_i) \propto -\frac{T}{2}\ln|\mathbf{S}_{i,xx}| - \frac{T}{2}\sum_{j=1}^{r_i}\ln\left(1 - \hat{\lambda}_{ij}\right). \tag{A.20}$$

Note that the maximized value of the log-likelihood $\ell_T^c(r_i)$ is only a function of the cointegration rank r_i , k_i and $k^* + s$ through the eigenvalues $\{\hat{\lambda}_{ij}\}_{j=1}^{r_i}$ defined by (A.19).[39]

The task of selecting the cointegrating rank, r_i, for each country/region can now be carried out using the log-likelihood ratio testing procedure. For example, to test the null hypothesis of cointegration rank r_i, H_{r_i} in (A.7), against the alternative hypothesis

$$H_{r_i+1}: \qquad Rank[\boldsymbol{\Pi}_i] = r_i + 1,\ r_i = 0, ..., k_i - 1,$$

[38] On the problem of identification of $\boldsymbol{\beta}_i$ see Pesaran and Shin (2001).

[39] For further details see Pesaran, Shin and Smith (2000). The necessary computations can be carried out using the econometrics software package Microfit 4.0. (see Pesaran and Pesaran (1997)). A less technical discussion of the various issues involved in the estimation of cointegrating/reduced rank regressions can also be found in Pesaran and Smith (1998).

the relevant log-likelihood ratio statistic is given by

$$\mathcal{LR}(H_{r_i}|H_{r_i+1}) = -T\ln(1-\hat{\lambda}_{i,r+1}), \tag{A.21}$$

where $\hat{\lambda}_{ir}$ is the r-th largest eigenvalue from the determinantal equation (A.19), $r_i = 0,...,k_i - 1$. This is usually referred to as the "maximum eigenvalue" statistic. To test the null hypothesis of cointegration rank r_i, H_{r_i}, $r_i = 0,...,k_i - 1$, against the alternative hypothesis of stationarity; that is

$$H_{k_i} : Rank[\mathbf{\Pi}_i] = k_i,$$

the appropriate test statistic is given by

$$\mathcal{LR}(H_{r_i}|H_{k_i}) = -T\sum_{j=r_i+1}^{k_i} \ln(1-\hat{\lambda}_{ij}). \tag{A.22}$$

This is known as the "trace statistic". Both of the above statistics have non-standard asymptotic distributions. But their critical values are tabulated in Pesaran, Shin and Smith (2000).[40] In small samples cointegration tests (the maximum eigenvalue as well as the trace tests) tend to have low power and consequently yield a cointegrating rank which is lower than the true value. Therefore, in practice it is advisable to use 90 per cent rather than the more usual 95 per cent level when carrying out the cointegration tests. Also the trace statistic tends to perform better than the maximum eigenvalue test.

Having chosen the cointegrating rank, r_i, the estimation of $\boldsymbol{\beta}_i$ can be carried out once suitable identifying and possibly over-identifying restrictions are imposed on the elements of $\boldsymbol{\beta}_i$. As was note above the solution to the eigenvalue problem, (A.19), only identifies $\boldsymbol{\beta}_i$ up to an $r_i \times r_i$ non-singular matrix. To investigate the necessary identification condition in the present application partition $\boldsymbol{\beta}_i$ as

$$\boldsymbol{\beta}_i = (\boldsymbol{\beta}'_{ix},\ \boldsymbol{\beta}'_{ix^*},\ \boldsymbol{\beta}'_{id},\ \boldsymbol{\beta}'_{i\iota})',$$

conformable to $\mathbf{v}_{it} = (\mathbf{x}'_{it}, \mathbf{x}^{*\prime}_{it}, \mathbf{d}'_t, t)'$. Then

$$\boldsymbol{\beta}'_i\mathbf{v}_{it} = \boldsymbol{\beta}'_{ix}\mathbf{x}_{it} + \boldsymbol{\beta}'_{ix^*}\mathbf{x}^*_{it} + \boldsymbol{\beta}'_{id}\mathbf{d}_t + \boldsymbol{\beta}'_{i\iota}t.$$

To identify $\boldsymbol{\beta}_i$ it is only necessary that $\boldsymbol{\beta}_{ix}$ (an $k_i \times r_i$ matrix), namely the part of $\boldsymbol{\beta}_i$ which corresponds to the endogenous variables, $\mathbf{x}_{it}$, are identified. For this purpose we need a total of r_i^2 restrictions; r_i restrictions on each of the r_i rows of $\boldsymbol{\beta}_{ix}$. Notice that in the stationary case where $r_i = k_i$ the identification of the long run relations involves setting $\boldsymbol{\beta}_{ix} = \mathbf{I}_{k_i}$. In cases where $r_i < k_i$, exact identification of $\boldsymbol{\beta}_i$ can be achieved by setting $\boldsymbol{\beta}'_{ix} = (\mathbf{I}_{r_i} \vdots \mathbf{Q}_i)$, where $\mathbf{Q}_i$ is an $r_i \times (k_i - r_i)$ matrix of fixed coefficients to be estimated freely. Other types of identifying restrictions, based on *a priori* economic theory can also be entertained. But all exactly identifying restrictions yield the same estimate of $\mathbf{\Pi}_i$, and hence for forecasting and impulse response analysis the results will be invariant to the choice of exact identifying restrictions. In what follows we suggest using the exact identifying restrictions $\boldsymbol{\beta}'_{ix} = (\mathbf{I}_{r_i} \vdots \mathbf{Q}_i)$, which are relatively simple to implement.

[40] Pesaran, Shin and Smith (2000) consider five different cases. It is their cases III and IV which are likely to be relevant to our particular application. See Section 9.

Finally, it may also be desirable to test the restrictions that all or some of the elements of $\boldsymbol{\beta}_{ix^*}$, $\boldsymbol{\beta}_{id}$ and $\boldsymbol{\beta}_{i\iota}$ are equal to zero. For example, while it may be reasonable to expect *changes* in oil prices to influence output changes, but it may be desirable to impose the restriction $\boldsymbol{\beta}_{id} = 0$. Similarly one can impose the co-trending restrictions $\boldsymbol{\beta}_{i\iota} = 0$. Clearly, it is also possible to test the validity of such restrictions by log-likelihood ratio tests.[41] Having estimated $\boldsymbol{\beta}_i$ the remaining parameters can be estimated along the lines suggested above. See (A.12) to (A.16).

[41] Microfit 4.0 allows the imposition and testing of over-identifying restrictions on $\boldsymbol{\beta}_i$. For further details see the lessons on cointegration analysis in Pesaran and Pesaran (1997, Ch. 16).

B Data Appendix

B.1 Data Sources

The primary variables (disaggregated by country/region when applicable) used in this study are:

Y :	Gross Domestic Product (GDP)
P :	General Price Index
Q :	Equity Price Index
E :	Exchange Rate
R :	Interest Rate
M :	Money Supply
PO :	Oil Price

B.1.1 Output (GDP)

The source for all 29 countries is the IFS GDP (1990) series. France, Germany, Italy, Japan, Mexico, the Netherlands, Spain, Switzerland, UK and USA are all from series BR, and the remaining countries are from series BP.

Where quarterly data were not available (ie, for Brazil, China, Indonesia, Kuwait, Malaysia, Poland, Saudi Arabia, Singapore, Thailand and Turkey), quarterly series were interpolated linearly from the annual series. For Singapore, Malaysia and Thailand, interpolated series were used only during the periods 1979-1992, 1979-1996 and 1979-1995, respectively. Quarterly output series were available for the subsequent periods. Data for the Czech Republic were available from 1992Q4 only.

For the period before German reunification, in 1990Q4, West German growth rates were used. The growth rate from 1988Q3 to 1990Q3 was used to compute a 'unified' output series for 1990Q4.

The data for Kuwait and Peru were rebased to 1990 using CPI for those countries, and the nominal GDP for Poland was deflated to real (1990 base) using CPI.

The data for Argentina and Singapore were seasonally adjusted.

B.1.2 General Price Indices

The data source for all countries except China was the IFS Consumer Price Index Series '64'. A full sample was available for all countries except Brazil, where 1979 data was unavailable, and a backcast using the average growth rate of prices for 1980 was employed.

B.1.3 Equity Price Indices

There were no data for China, Czech Republic, Poland, Russia or Saudi Arabia. For Austria we used Morgan Stanley Capital International (MSCI) series.

For Belgium, Indonesia, Italy, Malaysia, Singapore, Spain, Switzerland, Thailand, and Turkey we used Datastream, using quarterly averages from daily observations. However, we used quarterly average of weekly datapoints, as opposed to daily observations, for Argentina. The data for Malaysia was market cap weighted.

We used IFS data for Brazil, Chile, France, Germany, Japan, Korea, Mexico, The Netherlands, Peru, Philippines, UK and USA. Indices for share prices (IFS code "62") generally related to common shares of companies traded on national or foreign stock exchanges. Monthly indices were obtained as simple arithmetic averages of the daily or weekly observations("ZF").

These nominal equity price indices were deflated by the non-seasonally adjusted general price indices. The resultant real series were then adjusted for(possible) seasonal variations.

B.1.4 Exchange Rates

IFS series 'rf' was used for all countries. Data for the Czech Republic was available only from 1993Q1. The data was the period average using the official or principal rate

B.1.5 Interest Rates

Interest Rate data was taken from IFS Series '60B', the money market rate, with the following exceptions: for Argentina, Chile, China, Saudi Arabia and Turkey we used the IFS deposit rate; for the Czech Republic and Peru we used the IFS discount rate; for Hungary and the Philippines we used the IFS Treasury rate; and for Poland we used the IFS lending rate. Data for the Czech Republic was available only from 1993Q1.

B.1.6 Money Supply

The Money Supply data source for all countries was the sum of IFS series 34 (money) and series 35 (quasi-money). All series were seasonally adjusted. The data for Argentina, Brazil, Peru and Turkey required a decimal place adjustment to make the Money:GDP ratio reasonable.

For Belgium, we used quarterly data for all quasi-money; for money we used annual data converted to quarterly through interpolation up to 1990, and quarterly data from 1990Q4 to 1999Q1.

There were no data for Hungary from 1979-1982. From 1982-1987 we used annual data converted to quarterly through interpolation. Quarterly data were available from 1987Q4 to 1999Q1.

We used annual data converted to quarterly through interpolation for the Philippines and Poland, for the Philippines this was necessary for the period 1984-1986 only since quarterly data were available thereafter. There were no quarterly data for Saudi Arabia for 1983, and therefore annual data were used for that year.

Data for the Czech Republic were available only from 1993Q1.

B.1.7 Oil Price Index

For oil prices we used monthly averages of Brent Crude series from Datastream.

B.2 Construction of Regional Data Series

Time series observations at the regional level were constructed as weighted averages of corresponding country-specific series as set out in equations (8.10) and (8.11). For weights we used the GDP shares of each country in the region, computed as that country's PPP-adjusted GDP divided by the total PPP/USD GDP of the region. In order to avoid the use of time-varying weights, we choose a relatively recent time period for which PPP data is available, namely 1996.

Not all time series were available for all countries over the entire sample period. As a result the composition of the regional series is allowed to change as data on specific countries become available. For example, if data is not available for a given country over the first few periods in the sample, a zero weight is attached to this country with the weights of the remaining countries in the region adjusted to ensure that the sum of the weights add up to unity. Once data becomes available for the country in question, the weights are redistributed and the new information is 'folded into' the dataset.

Foreign variables are constructed uniquely for each region. For example, foreign money supply m^* for Western Europe is different from m^* for Latin America. We use the trade shares to appropriately weight the influence of foreign regions on a specified region's economy. Using an inter-regional trade matrix, we first compute the trade shares for each region with a given country (eg. the percent of Argentina's trade originating from the Western Europe), and then aggregate across countries based on the trade weights of the countries within the region.

The weights used to aggregate, across countries, the foreign variables need to be constructed with care. Since each *starred* variable is a weighted average of regional *starred* variables, if a given region's x variable is not available, then the weighted average must be adjusted to reflect the fact that the foreign variable is not comprised of all the x variables. This can easily be accomplished. For example, suppose that we are computing the German $q^*$and that x% of Germany's trade is with Turkey. However, Turkey's equity index is not available. When we take a weighted average of Germany's trading partners' equity indices, we will be effectively only weighting (1-x)%, since the Turkish index is unavailable. We can then divide our result by (1-x)% to yield the appropriate q^* for Germany. Finally, for regions with more than one member country, there exists 'intra-regional' trade (ie. trade between countries in the same region) that will not appear in the 'foreign' (*starred*) variables. As such, the weights will sum to less than one.

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Figure 1
Impulse Response of a Negative Unit (-1 SE) Shock to US Real Equity Prices on Real Equity Prices Across Regions

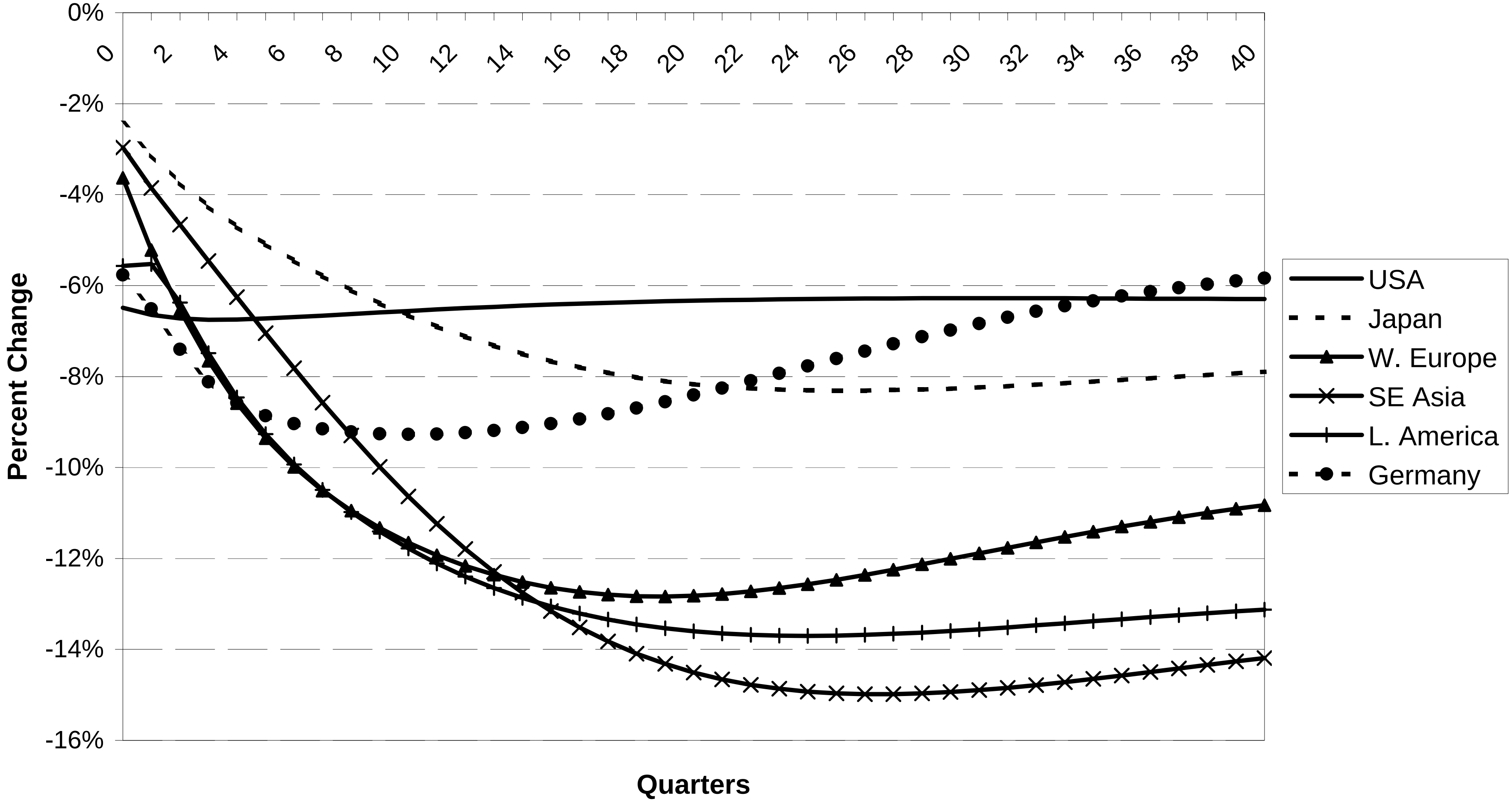

Figure 2
Impulse Response of a Negative Unit (-1 SE) Shock to US Real Equity Prices on Real Output Across Regions

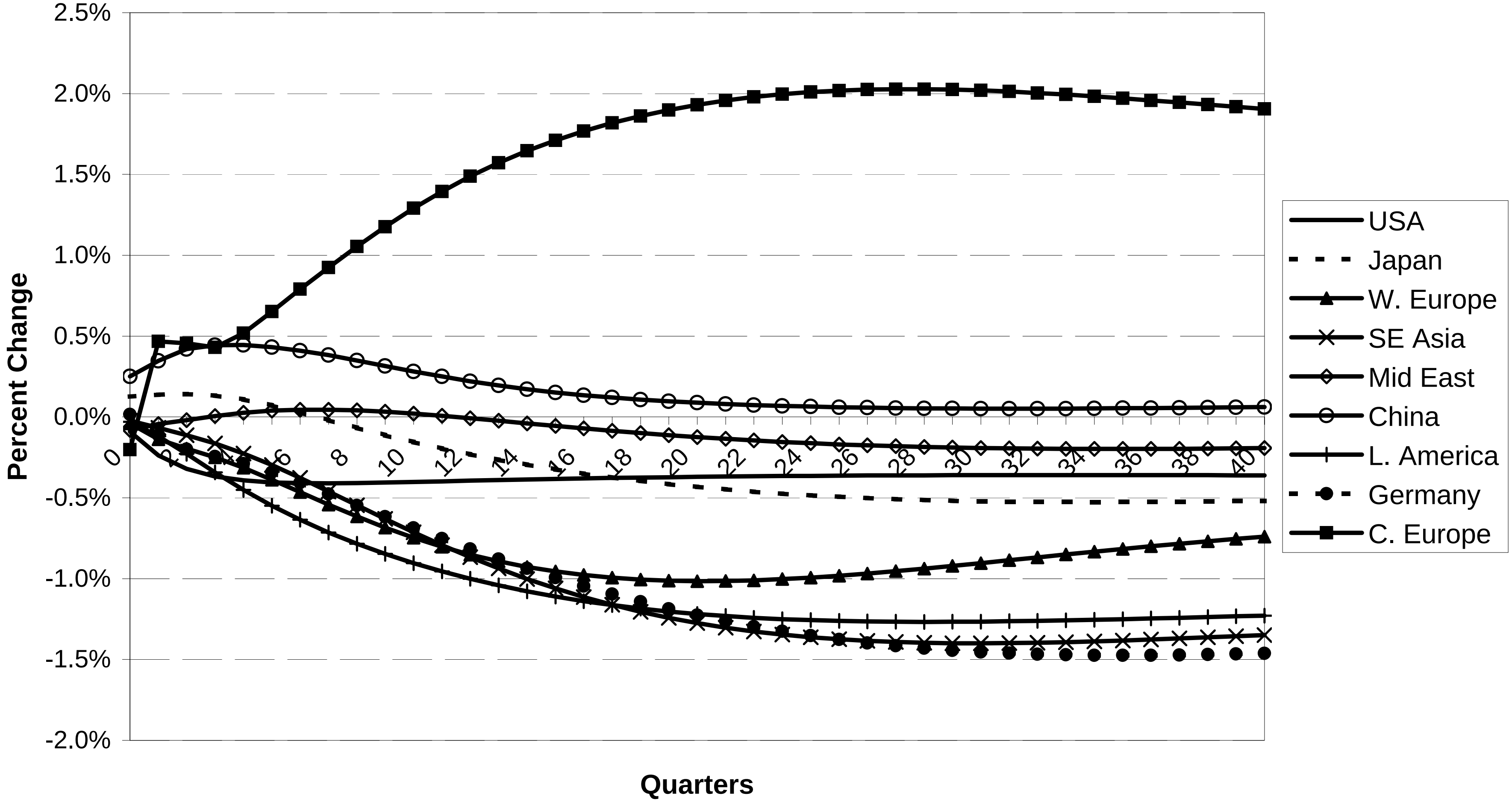

Figure 3
Impulse Response of a Positive Unit (+1 SE) Shock to US Interest Rates on Real Equity Prices Across Regions

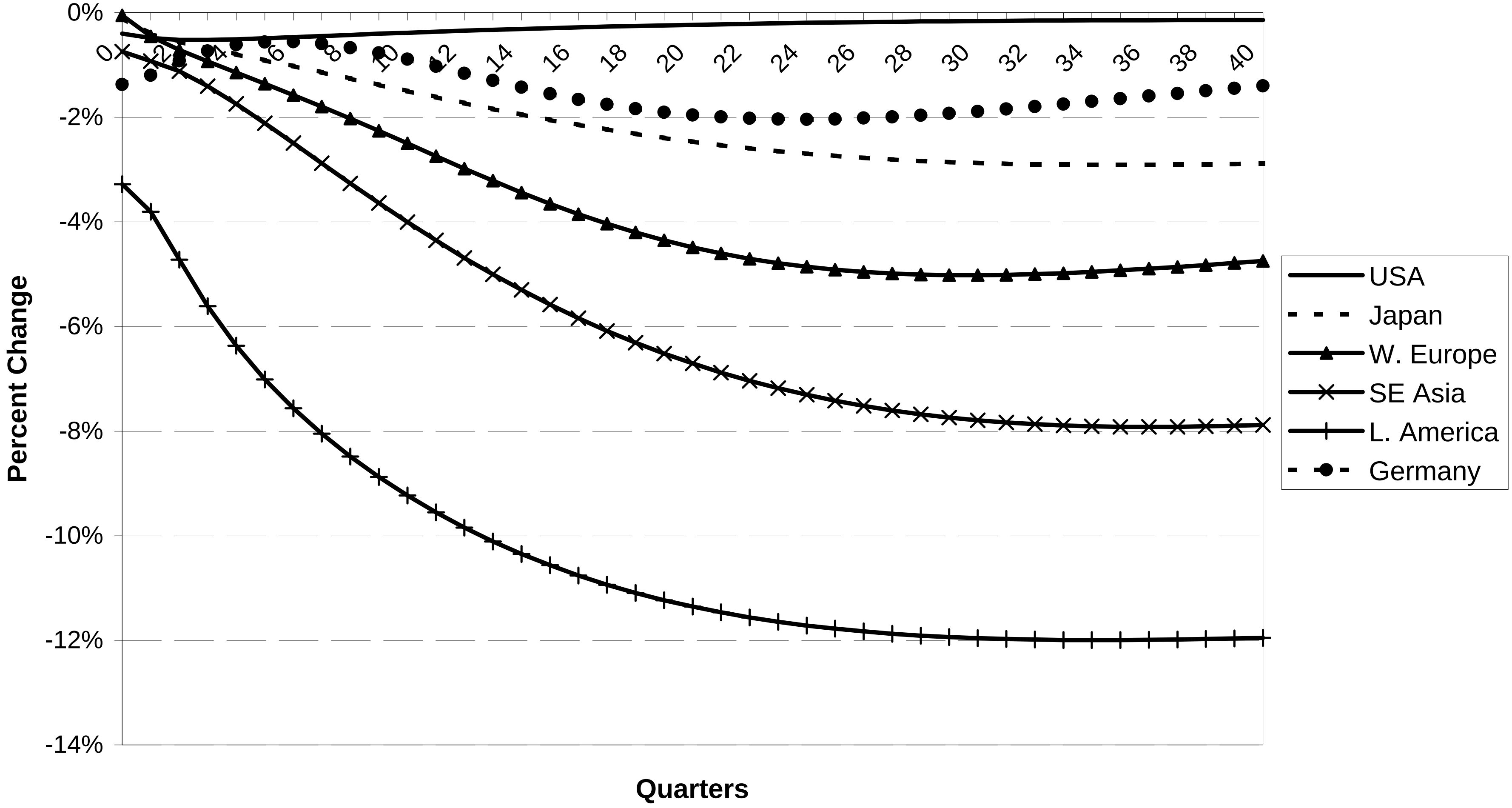

Figure 4
Impulse Response of a Positive Unit (+1 SE) Shock to US Interest Rates on Real Output Across Regions

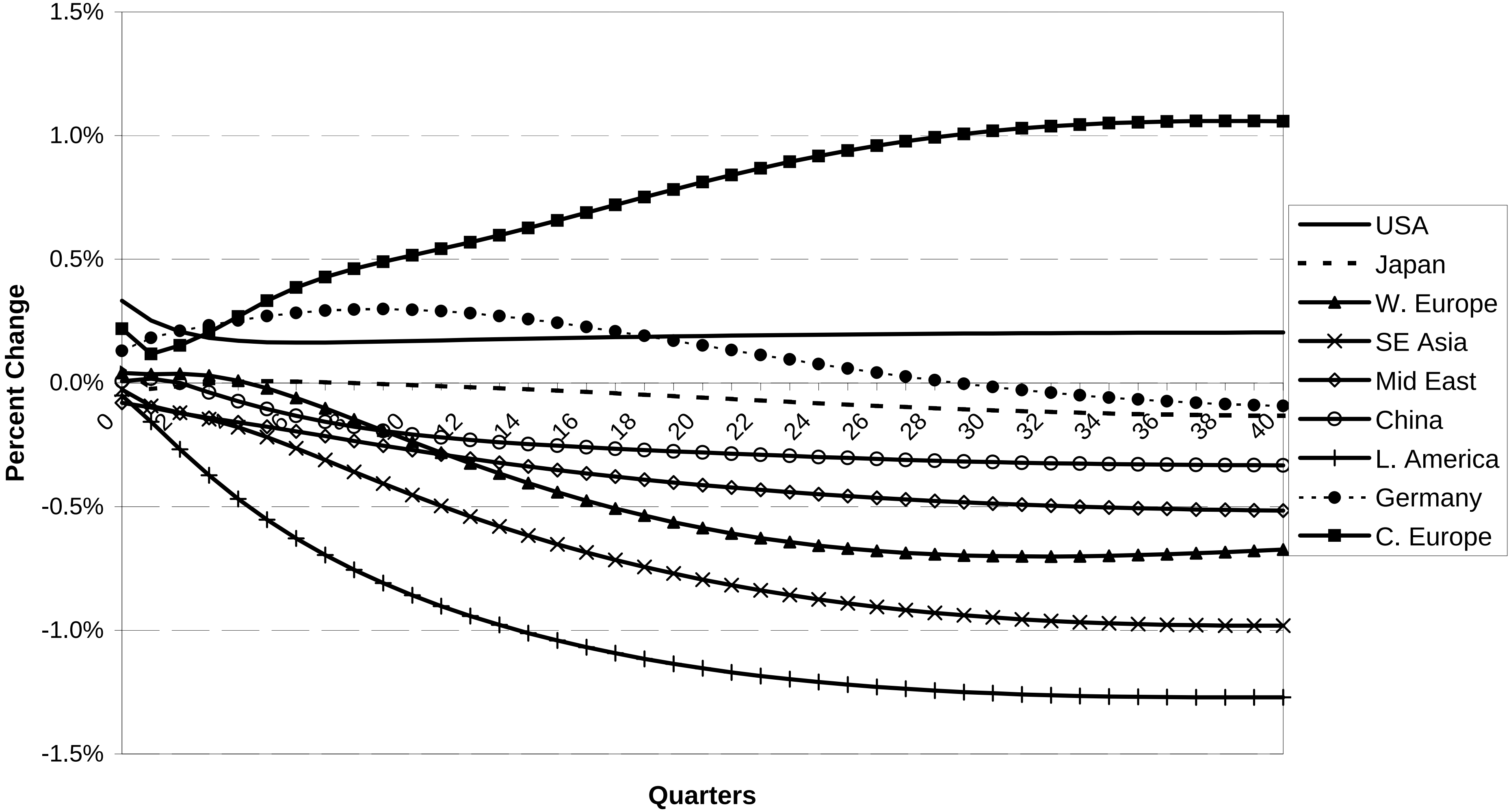

Figure 5
Impulse Response of a Negative Unit (-1 SE) Shock to Equity Markets in SE Asia on Real Equity Prices Across Regions

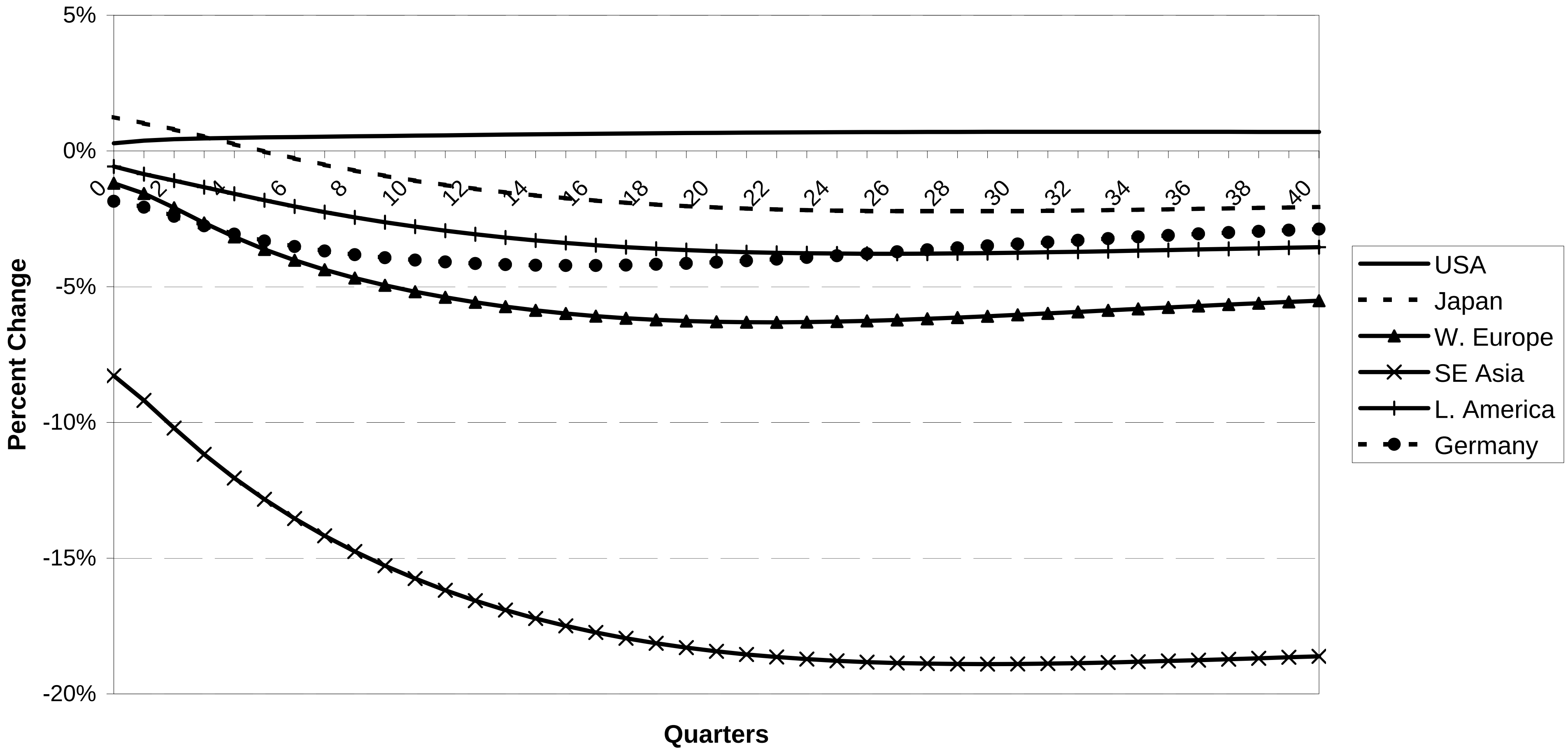

Figure 6
Impulse Response of a Negative Unit (-1 SE) Shock to Equity Markets in SE Asia on Real Output Across Regions

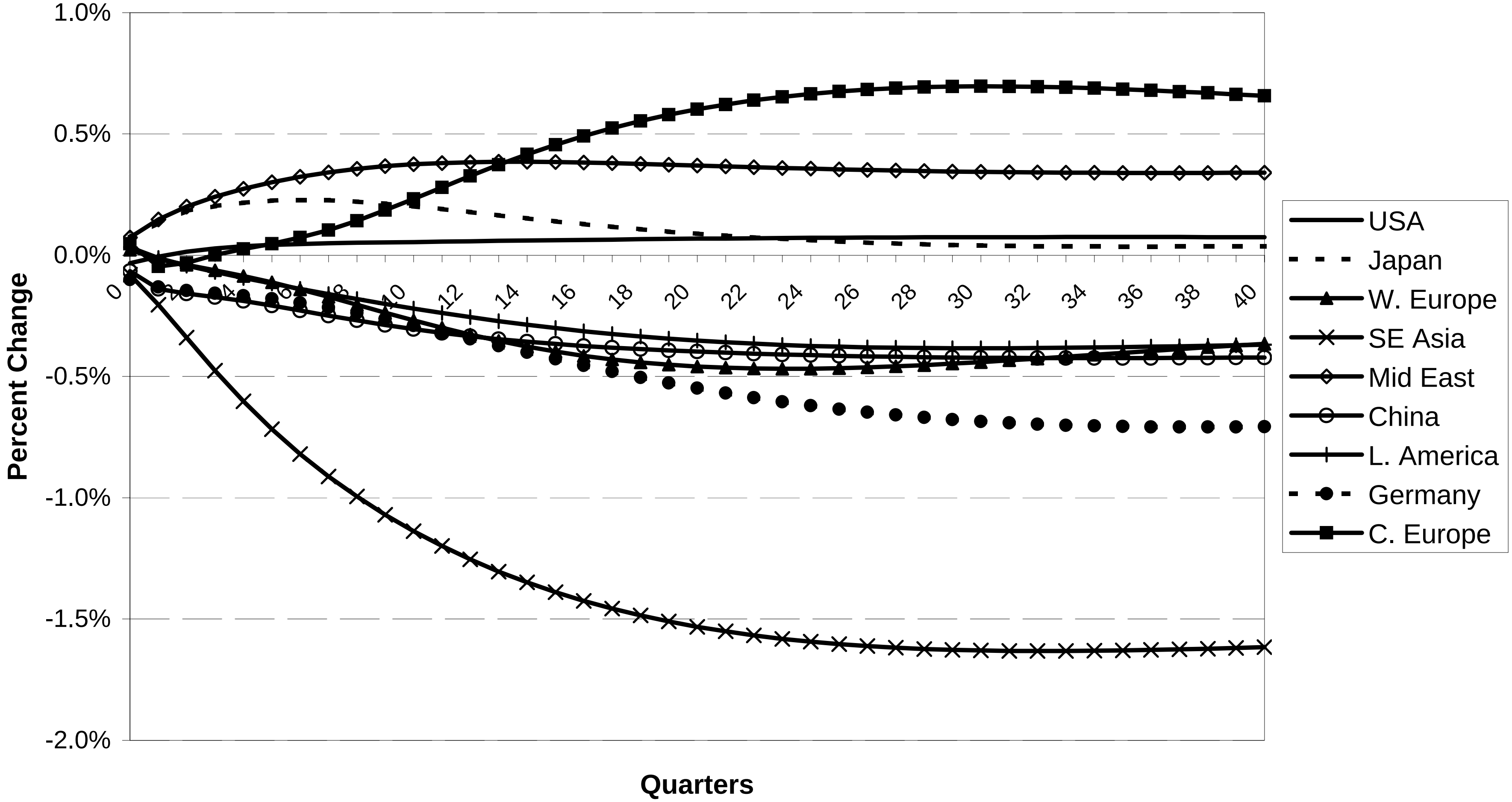

Table 2

Trade Weights Based on Direction of Trade Statistics[1]

Country/Region	USA	Western Europe	Germany	Japan	South East Asia	Latin America	China	Middle East	Central Europe
USA	0.0000	0.2221	0.1145	0.3517	0.3242	0.6781	0.2356	0.1962	0.0566
Western Europe	0.2247	0.0000	0.6478	0.1270	0.1688	0.1437	0.1364	0.3297	0.3737
Germany	0.0805	0.4499	0.0000	0.0599	0.0632	0.0493	0.0609	0.1191	0.4904
Japan	0.2007	0.0716	0.0487	0.0000	0.2813	0.0569	0.2922	0.1487	0.0173
South East Asia	0.1815	0.0934	0.0504	0.2760	0.0000	0.0418	0.2250	0.1682	0.0343
Latin America	0.2269	0.0475	0.0235	0.0334	0.0250	0.0000	0.0315	0.0167	0.0085
China	0.0506	0.0290	0.0186	0.1100	0.0864	0.0202	0.0000	0.0174	0.0132
Middle East	0.0294	0.0488	0.0254	0.0390	0.0449	0.0075	0.0121	0.0000	0.0060
Central Europe	0.0058	0.0377	0.0712	0.0031	0.0063	0.0026	0.0063	0.0041	0.0000

Source: International Trade Statistics Year Book, United Nations, 1996-98.

[1] Trade weights are computed as shares of exports and imports and displayed in columns by country/region

Table 3a

Augmented Dickey-Fuller (ADF) Unit Root Test Statistics

Domestic Variables	USA	Western Europe	Germany	Japan	South East Asia	Latin America	China	Middle East	Central Europe
y	-2.84	-2.28	-2.99	-0.29	-2.19	-3.29	-3.78	-3.10	-1.98
Δy	-4.36	-3.41	-3.09	-1.96	-4.27	-2.56	-2.69	-5.17	-4.07
$\Delta^2 y$	-6.80	-6.73	-10.57	-10.37	-2.75	-7.60	-5.20	-4.83	-5.65
p	-1.93	-1.25	-2.89	-2.49	-2.57	-1.53	-2.64	-1.50	-2.04
Δp	-4.30	-2.77	-1.90	-4.07	-3.67	-1.87	-2.05	-3.50	-2.76
$\Delta^2 p$	-6.88	-7.50	-8.90	-7.68	-6.58	-5.86	-4.83	-6.90	-6.63
e	-	-2.97	-2.96	-2.01	-1.87	-1.84	-6.28	-0.75	-1.66
Δe	-	-3.36	-3.37	-3.65	-4.39	-2.36	-4.31	-4.57	-4.06
$\Delta^2 e$	-	-7.66	-7.66	-7.92	-7.43	-5.30	-5.03	-7.43	-6.41
$e-p$	-	-2.32	-2.82	-1.74	-2.14	-1.97	-1.50	-1.95	-1.56
$\Delta(e-p)$	-	-3.77	-3.48	-3.76	-4.42	-2.44	-3.85	-4.21	-5.20
$\Delta^2(e-p)$	-	-7.70	-7.81	-7.91	-7.87	-7.07	-8.35	-8.28	-6.59
r	-1.79	-0.32	-2.46	-1.36	-2.85	-1.93	-1.93	-2.30	-1.69
Δr	-4.75	-5.07	-3.78	-7.49	-4.73	-5.33	-3.28	-4.58	-4.44
$\Delta^2 r$	-9.12	-8.26	-7.49	-7.07	-6.19	-9.00	-7.87	-10.30	-9.98
m	-2.05	-2.20	-2.91	-1.60	-2.43	-2.33	-3.30	-1.70	-1.61
Δm	-2.92	-2.92	-4.80	-2.45	-3.87	-3.63	-2.41	-2.28	-3.49
$\Delta^2 m$	-7.29	-10.43	-7.14	-7.51	-8.00	-9.69	-16.92	-8.00	-8.86
q	-1.47	-1.89	-2.63	-0.87	-1.55	-3.60	-	-	-
Δq	-4.43	-4.00	-3.78	-3.81	-3.08	-4.76	-	-	-
$\Delta^2 q$	-7.71	-7.02	-7.40	-6.63	-5.75	-7.00	-	-	-

Notes:
The ADF statistics are based on a univariate VAR model of order 5 in the level of the variables, and the statistics for the level, first differences and second differences of the variables are all computed on the basis of the same sample period, namelyu, 1980Q2 to 1999Q1
The ADF statistics for all the level variables are based on regressions including a linear trend, except for the interest variables.
The 95% critical value of the ADF statistics for regressions with trend is –3.47, and for regressions without trend -2.90

Table 3b

Augmented Dickey-Fuller (ADF) Unit Root Test Statistics

Foreign Variables	USA	Western Europe	Germany	Japan	South East Asia	Latin America	China	Middle East	Central Europe
y^*	-	-3.30	-2.92	-2.39	-1.95	-2.94	-1.26	-1.68	-2.89
Δy^*	-	-3.13	-3.27	-3.55	-3.21	-4.40	-2.48	-2.85	-3.01
$\Delta^2 y^*$	-	-9.02	-6.35	-5.70	-8.47	-6.69	-7.04	-7.24	-9.37
$*p$	-	-2.01	-1.15	-1.88	-1.03	-1.57	-1.00	-0.63	-0.99
Δp^*	-	-1.87	-1.94	-2.70	-2.22	-4.21	-2.67	-2.61	-1.95
$\Delta^2 p^*$	-	-6.28	-4.65	-5.94	-5.50	-6.24	-5.51	-5.78	-7.79
e^*	-1.99	-2.78	-1.65	-1.99	-2.15	-1.80	-1.99	-1.73	-1.74
Δe^*	-2.59	-3.57	-3.65	-3.58	-4.25	-2.39	-3.70	-4.40	-3.79
$\Delta^2 e^*$	-7.65	-6.59	-6.64	-7.90	-7.43	-5.16	-8.08	-7.81	-6.52
$e^*\text{-}p^*$	-2.11	-2.17	-0.62	-1.75	-1.98	-1.83	-2.45	-1.69	-1.85
$\Delta(e^*\text{-}p)^*$	-3.11	-3.24	-3.27	-3.38	-3.85	-2.45	-3.78	-3.93	-3.78
$\Delta^2(e^*\text{-}p)^*$	-9.10	-6.36	-6.40	-8.02	-7.50	-5.12	-8.02	-7.69	-6.56
r^*	-	-1.94	-1.25	-2.35	-1.27	-1.67	-2.27	-1.23	-1.32
Δr^*	-	-4.89	-4.32	-4.64	-4.76	-4.50	-4.80	-4.41	-4.09
$\Delta^2 r^*$	-	-9.19	-9.07	-9.45	-9.51	-9.11	-9.36	-9.11	-8.44
m^*	-	-2.91	-2.68	-2.15	-2.61	-2.24	-1.75	-2.21	-2.27
Δm^*	-	-3.64	-2.72	-2.91	-2.31	-2.73	-3.17	-2.84	-3.47
$\Delta^2 m^*$	-	-8.73	-10.84	-11.09	-11.95	-7.20	-7.04	-9.61	-8.88
q^*	-	-2.46	-1.89	-2.20	-1.65	-1.91	-1.33	-1.74	-2.23
Δq^*	-	-4.29	-4.27	-4.62	-4.42	-4.58	-4.37	-4.48	-3.96
$\Delta^2 q^*$	-	-7.40	-6.80	-6.57	-6.91	-7.41	-6.09	-6.52	-7.28

See the notes to Table 3a

Table 4a

Cointegration Rank Statistics for Regions with Capital Markets[1]

H_0	H_1	Western Europe	Germany	Japan	South East Asia	Latin America	Critical Values 95%	Critical Values 90%
Maximum Eigenvalue Statistics								
$r = 0$	$r = 1$	103.01	109.89	94.99	145.63	154.54	61.74	58.48
$r < 1$	$r = 2$	82.92	79.03	78.26	95.51	65.80	55.40	52.18
$r \leq 2$	$r = 3$	60.09	64.88	53.79	43.64	45.75	49.16	46.08
$r \leq 3$	$r = 4$	39.75	43.85	41.76	36.33	31.54	42.91	40.06
$r \leq 4$	$r = 5$	34.05	32.44	26.98	22.02	26.51	36.02	33.10
$r \leq 5$	$r = 6$	21.49	19.44	17.79	21.80	16.89	28.57	25.55
Trace Statistics								
$r = 0$	$r > 1$	341.30	349.53	313.56	364.93	331.03	191.45	184.80
$r < 1$	$r \geq 2$	238.29	239.65	218.58	219.30	148.50	152.15	145.67
$r \leq 2$	$r \geq 3$	155.38	160.61	140.31	123.79	120.70	115.43	110.31
$r \leq 3$	$r \geq 4$	95.28	95.73	86.52	**80.15**	**74.95**	83.43	78.85
$r \leq 4$	$r \geq 5$	55.34	**51.88**	**44.76**	43.82	43.40	54.21	50.39
$r \leq 5$	$r \geq 6$	**21.49**	19.44	17.79	21.80	16.89	28.57	25.55

1 The model contains unrestricted intercepts and restricted trend coefficients with I(1) endogenous variables: y, Δp, q, e-p, ρ, m, and I(1) exogenous variables: y*, Δp*, q*, ρ*, m*, p^0.

2 The bold statistics identify…

Table 4b

Cointegration Rank Statistics for Regions without Capital Markets[1]

H_0	H_1	China	Middle East	Central Europe	Critical Values 95%	Critical Values 90%
Maximum Eigenvalue Statistics						
$r = 0$	$r = 1$	50.77	72.06	104.75	55.40	52.18
$r < 1$	$r = 2$	46.68	61.56	73.81	49.16	46.08
$r \leq 2$	$r = 3$	37.76	51.14	48.40	42.91	40.06
$r \leq 3$	$r = 4$	28.90	30.85	43.34	36.02	33.10
$r \leq 4$	$r = 5$	18.13	18.10	17.23	28.57	25.55
Trace Statistics						
$r = 0$	$r > 1$	182.24	233.72	287.54	152.15	145.67
$r < 1$	$r \geq 2$	131.47	161.66	182.79	115.43	110.31
$r \leq 2$	$r \geq 3$	84.79	100.09	108.98	83.43	78.85
$r \leq 3$	$r \geq 4$	**47.03**	**44.95**	60.58	54.21	50.39
$r \leq 4$	$r \geq 5$	18.13	18.10	**17.23**	28.57	25.55

[1] See notes to Table 4a

Table 4c

Cointegration Rank Statistics for the US Model[1]

Maximum Eigenvalue Statistics				
			Critical Values	
H_0	H_1	**USA**	**95%**	**90%**
$r = 0$	$r = 1$	77.10	39.85	37.15
$r < 1$	$r = 2$	25.49	33.87	31.30
$r \leq 2$	$r = 3$	15.82	27.75	25.21
$r \leq 3$	$r = 4$	5.38	21.07	18.78
$r \leq 4$	$r = 5$	2.89	14.35	12.27
Trace Statistics				
$r = 0$	$r > 1$	126.70	92.42	87.93
$r < 1$	$r \geq 2$	**49.60**	68.06	63.57
$r \leq 2$	$r \geq 3$	24.10	46.44	42.67
$r \leq 3$	$r \geq 4$	8.28	28.42	25.63
$r \leq 4$	$r \geq 5$	2.89	14.35	12.27

[1] The model contains unrestricted intercepts but no deterministic trends with I(1) endogenous variables: y, Δp, q, ρ, m, and I(1) exogenous variables: e^*-p^*, p^0.

Table 5
In-Sample Root Mean Square Forecast Errors (in per cent)
(1979Q3-1999Q1)

Factor	Country/Region	GVAR Model	Benchmark Model[1]
Real Output	**USA**	0.701	0.790
	Western Europe	0.264	0.432
	Germany	0.807	0.926
	Japan	0.683	0.863
	South East Asia	0.502	1.100
	Latin America	0.586	0.769
	China	1.000	1.060
	Middle East	0.598	0.705
	Central Europe	2.590	3.480
Average across regions		*1.070*	*1.410*
Inflation	**USA**	0.467	0.507
	Western Europe	0.294	0.347
	Germany	0.367	0.510
	Japan	0.365	0.587
	South East Asia	0.743	1.130
	Latin America	7.750	9.200
	China	0.686	0.744
	Middle East	2.560	3.570
	Central Europe	6.750	7.740
Average across regions		*3.550*	*4.220*
Interest Rate	**USA**	0.269	0.265
	Western Europe	0.109	0.151
	Germany	0.092	0.139
	Japan	0.107	0.165
	South East Asia	0.386	0.450
	Latin America	10.600	10.900
	China	0.111	0.147
	Middle East	0.958	1.300
	Central Europe	0.932	1.190
Average across regions		*3.560*	*3.680*
Real Equity Price	**USA**	6.060	6.090
	Western Europe	7.080	6.930
	Germany	8.910	8.110
	Japan	7.230	7.760
	South East Asia	8.660	10.600
	Latin America	17.000	17.600
	China	-	-
	Middle East	-	-
	Central Europe	-	-
Average across regions		*9.860*	*10.300*

continued/….

Table 5 (continued)

In-Sample Root Mean Square Forecast Errors (in per cent) (1979Q3-1999Q1)

Real Exchange Rate	**USA**	-	-
	Western Europe	4.090	4.680
	Germany	4.590	5.020
	Japan	5.380	5.580
	South East Asia	4.310	5.650
	Latin America	4.160	4.830
	China	5.200	5.710
	Middle East	3.710	4.350
	Central Europe	5.080	6.370
Average across regions		*4.600*	*5.310*
Real Money Balances	**USA**	1.100	1.250
	Western Europe	1.630	1.730
	Germany	1.590	1.710
	Japan	0.851	1.220
	South East Asia	1.470	1.620
	Latin America	5.090	5.590
	China	4.030	4.590
	Middle East	0.965	1.170
	Central Europe	5.580	7.130
Average across regions		*3.040*	*3.590*

[1] The following random walk models with drifts were used as benchmarks:

$$y_{it} = y_{i,t-1} + \mu_i^y + \varepsilon_{it}^y, \Delta p_{it} = \Delta p_{i,t-1} + \mu_i^\pi + \varepsilon_{it}^\pi,$$

$$q_{it} = q_{i,t-1} + \mu_i^q + \varepsilon_{it}^q, r_{it} = r_{i,t-1} + \mu_i^r + \varepsilon_{it}^r,$$

$$e_{it} - \rho_{it} = e_{i,t-1} - \rho_{i,t-1} + \mu_i^e + \varepsilon_{it}^e, m_{it} = m_{i,t-1} + \mu_i^m + \varepsilon_{it}^m.$$

The drift parameters were estimated using the in-sample observations.

Table 6
Generalized Impulse Responses of a Negative One Standard Error Shock to US Equity Prices

Region	Quarters after Shock 0	1	2	3	4	8	12	20
	on equity prices (%)							
USA	-6.5	-6.6	-6.7	-6.7	-6.7	-6.6	-6.5	-6.3
Western Europe	-3.6	-5.2	-6.5	-7.7	-8.6	-10.9	-12.2	-12.8
Germany	-5.8	-6.5	-7.4	-8.1	-8.6	-9.2	-9.2	-8.4
Japan	-2.4	-3.1	-3.7	-4.3	-4.7	-6.1	-7.1	-8.2
South East Asia	-3.0	-3.9	-4.7	-5.4	-6.3	-9.3	-11.8	-14.5
Latin America	-5.5	-5.5	-6.4	-7.5	-8.5	-11.0	-12.4	-13.6
China	-	-	-	-	-	-	-	-
Middle East	-	-	-	-	-	-	-	-
Central Europe	-	-	-	-	-	-	-	-
	on output (%)							
USA	-0.09	-0.24	-0.32	-0.37	-0.39	-0.41	-0.39	-0.37
Western Europe	-0.04	-0.14	-0.20	-0.25	-0.31	-0.62	-0.85	-0.10
Germany	0.01	-0.11	-0.20	-0.25	-0.29	-0.55	-0.82	-0.12
Japan	0.13	0.14	0.14	0.13	0.11	0.06	-0.23	-0.43
South East Asia	-0.02	-0.07	-0.11	-0.16	-0.23	-0.55	-0.87	-1.28
Latin America	-0.03	-0.12	-0.23	-0.34	-0.45	-0.78	-1.00	-1.22
China	0.25	0.35	0.42	0.44	0.45	0.35	0.22	0.09
Middle East	-0.08	-0.04	-0.02	0.00	0.03	0.04	0.00	-0.12
Central Europe	-0.20	0.47	0.46	0.43	0.52	1.05	1.45	1.93
	on inflation (%)							
USA	0.11	0.01	-0.04	-0.07	-0.08	-0.08	-0.08	-0.06
Western Europe	0.59	0.69	0.08	0.07	0.07	0.04	0.63	-0.05
Germany	0.02	0.76	-0.01	-0.03	-0.39	-0.08	-0.12	-0.17
Japan	-0.06	-0.03	-0.51	-0.06	-0.07	-0.10	-0.13	-0.16
South East Asia	-0.05	-0.59	-0.09	-0.10	-0.10	-0.13	-0.18	-0.26
Latin America	1.22	1.59	1.09	1.04	0.39	1.60	1.89	2.17
China	0.03	0.08	0.15	0.19	0.22	0.31	0.32	0.29
Middle East	0.72	0.39	0.42	0.35	0.30	0.08	-0.10	-0.29
Central Europe	-0.29	-0.71	-1.09	-1.11	-0.99	-0.08	-0.86	-1.64
	on interest rate (%)							
USA	0.02	0.19	-0.74	-0.01	-0.01	-0.02	-0.01	-0.01
Western Europe	0.49	0.12	0.26	0.40	0.26	-0.01	-0.03	-0.06
Germany	0.23	-0.01	-0.03	-0.06	-0.08	-0.15	-0.23	-0.36
Japan	0.02	0.02	0.02	0.01	0.79	-0.01	-0.02	-0.06
South East Asia	-0.02	0.88	0.01	0.02	0.02	0.03	0.02	0.77
Latin America	1.75	1.35	1.19	1.27	1.42	1.89	2.19	2.47
China	0.02	-0.22	0.92	0.02	0.03	0.06	0.07	0.06
Middle East	0.21	0.17	0.24	0.27	0.31	0.44	0.54	0.65
Central Europe	-0.18	-0.20	-0.20	-0.23	-0.24	-0.29	-0.43	-0.71
	on real exchange rate (%)							
USA	-	-	-	-	-	-	-	-
Western Europe	-0.54	0.33	0.93	1.45	1.92	3.44	4.44	5.24
Germany	-1.63	-1.70	-1.89	-1.97	-1.90	-1.05	0.08	2.39
Japan	-0.37	-0.32	-0.44	-0.48	-0.46	-0.19	0.10	0.49
South East Asia	0.37	0.12	0.07	0.09	0.13	0.35	0.52	0.70
Latin America	0.52	0.74	1.03	1.41	1.79	3.00	3.82	4.66
China	1.01	0.35	-0.36	-0.89	-1.29	-2.25	-2.55	-2.48
Middle East	-0.58	-0.88	-1.21	-1.50	-1.71	-2.10	-2.08	-1.64
Central Europe	0.70	-0.74	-1.98	-2.92	-3.62	-5.00	-5.67	-6.93

Table 7
Generalized Impulse Responses of a Positive One Standard Error Shock to US Interest Rates

Region	Quarters after Shock							
	0	1	2	3	4	8	12	20
	on equity prices (%)							
USA	-0.40	-0.49	-0.52	-0.52	-0.51	-0.42	-0.34	-0.32
Western Europe	-0.05	-0.45	-0.72	-0.93	-1.15	-2.03	-2.98	-4.49
Germany	-1.37	-1.20	-0.92	-0.73	-0.62	-0.67	-1.16	-1.96
Japan	-0.11	-0.40	-0.56	-0.68	-0.80	-1.26	-1.73	-2.47
South East Asia	-0.75	-0.93	-1.12	-1.41	-1.74	-3.26	-4.69	-6.71
Latin America	-3.28	-3.80	-4.72	-5.61	-6.37	-8.49	-9.84	-11.38
China	-	-	-	-	-	-	-	-
Middle East	-	-	-	-	-	-	-	-
Central Europe	-	-	-	-	-	-	-	-
	on output (%)							
USA	0.33	0.25	0.21	0.18	0.17	0.16	0.17	0.19
Western Europe	0.41	0.35	0.37	0.03	0.92	-0.15	-0.32	-0.59
Germany	0.13	0.18	0.21	0.23	0.27	0.30	0.28	0.15
Japan	0.06	-0.02	-0.01	-0.26	0.49	-0.83	-0.02	-0.06
South East Asia	-0.03	-0.09	-0.12	-0.15	-0.18	-0.36	-0.54	-0.79
Latin America	-0.05	-0.16	-0.27	-0.37	-0.47	-0.75	-0.94	-1.15
China	0.69	0.02	0.68	-0.04	-0.07	-0.18	-0.23	-0.28
Middle East	-0.08	-0.10	-0.12	-0.14	-0.16	-0.23	-0.31	-0.41
Central Europe	0.22	0.12	0.15	0.20	0.27	0.46	0.57	0.81
	on inflation (%)							
USA	0.17	0.12	0.09	0.08	0.70	0.07	0.08	0.09
Western Europe	0.03	0.05	0.06	0.05	0.05	0.04	0.02	-0.11
Germany	0.01	0.04	0.03	0.03	0.02	-0.49	-0.01	-0.37
Japan	-0.04	0.06	0.03	0.34	0.30	0.21	0.87	-0.01
South East Asia	0.22	0.03	-0.04	-0.05	-0.06	-0.09	-0.13	-0.19
Latin America	-0.28	-0.78	-0.91	-0.88	-0.80	-0.54	-0.36	-0.11
China	0.09	0.16	0.19	0.19	0.20	0.21	0.22	0.22
Middle East	0.27	0.20	0.13	0.07	0.03	-0.08	-0.16	-0.26
Central Europe	-0.44	0.17	0.72	1.17	1.53	2.26	2.45	2.37
	on interest rate (%)							
USA	0.28	0.28	0.26	0.26	0.26	0.26	0.26	0.26
Western Europe	-0.88	0.01	0.03	0.08	0.05	0.08	0.08	0.07
Germany	0.81	0.04	0.05	0.07	0.08	0.11	0.12	0.10
Japan	-0.02	-0.59	0.14	0.21	0.26	0.04	0.05	0.05
South East Asia	0.11	0.08	0.07	0.08	0.08	0.09	0.09	0.07
Latin America	-0.49	-0.68	-0.69	-0.60	-0.51	-0.22	-0.01	0.26
China	-0.67	0.45	0.02	0.03	0.03	0.04	0.04	0.04
Middle East	0.05	0.05	0.05	0.05	0.05	0.08	0.12	0.20
Central Europe	-0.01	0.04	0.12	0.21	0.29	0.44	0.45	0.38
	on real exchange rate (%)							
USA	-	-	-	-	-	-	-	-
Western Europe	0.60	0.97	1.30	1.57	1.81	2.66	3.42	4.50
Germany	0.47	1.30	1.67	1.82	1.87	0.18	1.75	1.90
Japan	0.25	0.47	0.39	0.40	0.42	0.55	0.66	0.84
South East Asia	0.44	-0.40	-0.58	-0.59	-0.58	-0.54	-0.52	-0.47
Latin America	-0.48	-0.20	0.15	0.51	0.84	1.85	2.52	3.30
China	0.05	-0.41	-0.78	-0.97	-1.08	-1.25	-1.32	-1.45
Middle East	0.06	-0.31	-0.10	-0.12	-0.10	0.02	0.19	0.52
Central Europe	0.10	0.04	0.18	0.51	0.91	2.23	2.69	2.41

Table 8
Generalized Impulse Responses of a One Standard Error Negative Shock to South East Asia Equities

Region	Quarters after Shock							
	0	1	2	3	4	8	12	20
	on equity prices (%)							
USA	0.28	0.38	0.43	0.46	0.48	0.54	0.59	0.67
Western Europe	-0.12	-1.57	-2.09	-2.65	-3.17	-4.68	-5.57	-6.29
Germany	-1.85	-2.07	-2.41	-2.76	-3.06	-3.82	-4.14	-4.10
Japan	1.25	1.02	0.79	0.52	0.25	-0.72	-1.40	-2.08
South East Asia	-8.28	-9.19	-10.21	-11.18	-12.05	-14.75	-16.57	-18.40
Latin America	-0.57	-0.85	-1.09	-1.33	-1.58	-2.45	-3.07	-3.69
China	-	-	-	-	-	-	-	-
Middle East	-	-	-	-	-	-	-	-
Central Europe	-	-	-	-	-	-	-	-
	on output (%)							
USA	-0.03	-0.59	0.01	0.03	0.04	0.05	0.06	0.07
Western Europe	0.02	-0.02	-0.04	-0.06	-0.08	-0.20	-0.33	-0.46
Germany	-0.10	-0.13	-0.15	-0.16	-0.17	-0.24	-0.34	-0.55
Japan	0.07	0.13	0.18	0.20	0.22	0.22	0.18	0.09
South East Asia	-0.08	-0.20	-0.34	-0.47	-0.60	-0.99	-1.25	-1.53
Latin America	0.03	0.01	0.04	0.07	0.09	-0.18	-0.25	-0.35
China	-0.65	-0.14	-0.16	-0.17	-0.18	-0.27	-0.33	-0.40
Middle East	0.07	0.15	0.20	0.24	0.27	0.36	0.38	0.37
Central Europe	0.05	-0.05	-0.03	0.15	0.03	0.14	0.33	0.60
	on inflation (%)							
USA	-0.05	-0.03	-0.02	-0.79	-0.20	-0.79	0.01	0.02
Western Europe	0.03	0.24	0.02	0.02	0.02	0.01	-0.30	-0.03
Germany	0.03	-0.02	-0.02	-0.03	-0.34	-0.05	-0.07	-0.10
Japan	-0.09	-0.05	-0.54	-0.05	-0.06	-0.09	-0.11	-0.13
South East Asia	-0.14	-0.11	-0.09	-0.10	-0.13	-0.02	-0.03	-0.04
Latin America	-0.20	0.26	0.32	0.35	0.36	0.42	0.51	0.66
China	0.07	0.05	0.06	0.07	0.08	0.08	0.07	0.05
Middle East	0.07	0.04	0.18	0.17	0.15	0.52	-0.14	-0.28
Central Europe	-0.03	-0.40	-0.10	-0.18	-0.25	-0.31	-0.35	-0.66
	on interest rate (%)							
USA	0.88	0.01	0.01	0.02	0.02	0.02	0.02	0.02
Western Europe	-0.21	0.11	0.32	0.32	0.33	0.30	-0.29	-0.02
Germany	-0.03	-0.03	-0.03	-0.04	-0.05	-0.08	-0.11	-0.17
Japan	-0.02	-0.02	0.02	-0.61	0.26	0.01	0.55	-0.01
South East Asia	0.06	0.23	0.02	0.01	0.52	-0.01	-0.03	-0.05
Latin America	0.12	0.25	0.31	0.35	0.37	0.47	0.58	0.72
China	-0.79	-0.22	0.45	0.74	0.96	0.01	0.89	-0.11
Middle East	0.09	-0.13	-0.01	-0.10	-0.10	-0.05	-0.78	0.06
Central Europe	0.06	-0.26	-0.04	-0.06	-0.07	-0.12	-0.17	-0.03
	on real exchange rate (%)							
USA	-	-	-	-	-	-	-	-
Western Europe	-0.21	0.11	0.32	0.32	0.33	0.30	-0.29	-0.02
Germany	-0.03	-0.03	-0.03	-0.04	-0.05	-0.08	-0.11	-0.17
Japan	-0.02	-0.02	-0.02	-0.61	0.21	0.01	0.55	-0.01
South East Asia	0.06	0.23	0.02	0.01	0.52	-0.01	-0.03	-0.05
Latin America	0.12	0.25	0.31	0.35	0.37	0.47	0.58	0.72
China	-0.79	0.22	0.45	0.74	0.96	0.01	0.89	-0.11
Middle East	0.09	-0.13	-0.11	-0.10	-0.10	-0.05	-0.78	0.06
Central Europe	0.06	-0.26	-0.04	-0.06	-0.07	-0.12	-0.17	-0.31